Trig Identities

Trig identities are useful facts about trig functions that are used to simplify things. There are many, many trig identities. Here we will talk about the most common and useful ones. Here is probably the most useful one of all:

$$\sin^2 x + \cos^2 x = 1.$$

Why is it true? Remember that cosine and sine are the x- and y-coordinates of points on the unit circle. You might remember that the equation of the unit circle is $x^2 + y^2 = 1$, and plugging cosine and sine in for x and y gives $\cos^2 x + \sin^2 x = 1$. It is also closely related to the Pythagorean formula.

If you divide through by $\sin^2 x$ or by $\cos^2 x$ and rearrange terms you get two other useful identities:

$$\sec^2 x + 1 = \tan^2 x \qquad \csc^2 x + 1 = \cot^2 x.$$

Here are two occasionally useful identities:

$$\sin(x + y) = \sin x \cos y + \cos x \sin y \qquad \cos(x + y) = \cos x \cos y - \sin x \sin y$$

Note in particular that you *can't* break up $\sin(x + y)$ into $\sin(x) + \sin(y)$. Here are two other occasionally useful identities:

$$\sin(-x) = -\sin(x) \qquad \cos(-x) = \cos(x)$$

And a few more:

$$\sin(2x) = 2 \sin x \cos x \qquad \cos(2x) = \cos^2 x - \sin^2 x$$

All of these can be found on the inside cover of most calculus books or in online reference sheets. Unless your professor wants you to memorize them, it is usually not worth memorizing them. But it is important to remember that they exist and to know where to find them when you need them.

Exercises

1. Simplify the expressions below as much as possible.

   (a) $\dfrac{(\sec x \tan x)[(1+\sec x)-\sec x)]}{1+\tan^2 x}$

   (b) $\dfrac{(\sin x)(-\sin x)-(\cos x)(\cos x)}{\sin^2 x}$

2. Show the following are true.

   (a) $\dfrac{\sin 2x}{2\cos x} = \sin x$

   (b) $\sin^2(x)+\cos^2(-x)=1$

3. Use the identity given for $\sin(x+y)$ to show that $\sin(2x) = 2\sin x \cos x$.

Answers

1. (a) Start by simplifying $(1 + \sec x) - \sec x$ into 1 in the numerator, and rewrite $1 + \tan^2$ in the denominator as $\sec^2 x$. Then the overall expression becomes $\frac{\sec x \tan x}{\sec^2 x}$. Cancel the $\sec x$ in the numerator with one in the bottom to get $\frac{\tan x}{\sec x}$. Rewrite $\tan x$ as $\sin x / \cos x$ and rewrite $\sec x$ as $1/\cos x$. Then the expression becomes $\frac{\sin x/\cos x}{1/\cos x}$. Multiply top and bottom by $\cos x/1$ to clear the complex fractions, and the entire expression simplifies down to $\sin x$.

   (b) Rewrite the numerator as $-(sin^2 x + \cos^2 x)$, which simplifies to -1 using a trig identity. Then the expression becomes $-1/sin^2 x$, which we could also write as $-csc^2 x$.

2. (a) Use a double-angle identity to rewrite the numerator as $2 \sin x \cos x$. Cancel the 2 and the $\cos x$ from the numerator and denominator, and we are left with $\sin x$, which is the right side.

   (b) Use the fact that $\cos -x = \cos x$ to rewrite the left side as $\sin^2 x + \cos^2 x$. A trig identity tells us that this is 1, which is what the right side is.

3. The rule is $\sin(x + y) = \sin x \cos y + \cos x \sin y$. Suppose we take $y = x$ in this rule. Then we have $\sin(x + x) = \sin x \cos x + \cos x \sin x$. Simplifying this gives $\sin(2x) = 2 \sin x \cos x$, as desired.