

The idea behind solving equations

Solving equations is one of the most common uses of algebra in calculus. Sometimes we are solving an equation for one variable in terms of others. Most often, we are solving an equation to find zeroes (or roots), which means we are looking for the values that make an expression equal to 0.

How to solve equations

The main idea is to use algebra to transform the equation until the solution appears. Usually this algebra involves adding or subtracting the same thing to both sides of an equation, multiplying or dividing both sides by something, or raising both sides to a power. The key idea is that whatever we do to one side of the equation, we have to do to the other side.

Factoring also is very useful, especially when an equation is set equal to 0. Once an equation is factored into a form like $AB = 0$ (where A and B can be expressions), then we can get the solutions by setting $A = 0$ and $B = 0$ separately. The reason this works is that if AB equals 0, then the only way that can happen is if either A or B is 0 (since the only way a product can be 0 is if one of its parts is 0).

Geometric and solving equations

Here are a few important facts:

- Anytime you want to find where two curves intersect, you can do so by setting their equations equal to each other and solving. The solutions tell us the points the curve intersect at. If there is no solution, then the curves don't meet.
- Anytime you want to find where a curve meets the x -axis (i.e., where it is 0), you can do so by setting the curve's formula equal to 0 and solving. If there is no solution, then the curve doesn't meet the x -axis and hence is never 0.

Exercises

1. Find where the curves $y = x$ and $y = x^2$ intersect.
2. Find where the curve $y = x^2 - 9$ meets the x -axis.

Answers

1. To find the intersection, set $x = x^2$. Move both terms to the same side to get $x^2 - x = 0$. Factor to get $x(x - 1) = 0$. Set the parts equal to 0 separately to get $x = 0$ and $x = 1$.
2. It meets the x when $0 = x^2 - 9$. Simplify this to get $x^2 = 9$ and so $x = \pm 3$.