Solving basic trig equations

Solving trig equations can get pretty complicated pretty fast, but here will will just cover the basics.

Example 1 Find all solutions of $\sin(3x) = 0$.

Solution: The starting point is that the sine function is 0 at all multiples of π. Think about it as $\sin(*) = 0$ whenever $*$ is a multiple of π. Here $*$ is $3x$, so the solution comes from setting $3x$ equal to multiples of π.

To start, we have $3x = 0$, which gives $x = 0$. Next we have $3x = \pi$, which gives $x = \pi/3$. Then we have $3x = 2\pi$, which gives $x = \frac{2\pi}{3}$. Then we have $3x = 3\pi$, which gives $x = \pi$. Then $3x = 4\pi$, which gives $x = \frac{4\pi}{3}$. From here we see the general pattern that the solutions are all of the form $x = \frac{n\pi}{3}$, where n can be any integer, positive or negative.

Example 2 Find all solutions to $7\cos(4x + 1) = 0$.

Solution: First divide both sides by 7 to get $\cos(4x + 1) = 0$. Next we need to know when cosine is 0. This happens at odd multiples of $\pi/2$. We then set the inside of the cosine expression equal to these multiples.

To start, we have $4x + 1 = \pi/2$. This gives $x = (\pi/2 - 1)/4$. Then we do $4x + 1 = 3\pi/2$ and get $x = (3\pi/2 - 1)/4$. Next we do $4x + 1 = 5\pi/2$ and get $x = (5\pi/2 - 1)/4$. In general, all solutions are of the form $(n\pi/2 - 1)/4$, where n is any odd integer, positive or negative.

Exercises

Solve the following equations.

1. $\cos(5x) = 0$
2. $\sin(2x + 1) = 0$

Answers

1. We know that cosine is 0 at odd multiples of $\pi/2$, namely $\pm\pi/2, \pm 3\pi/2, \pm 5\pi/2$, etc. Set $5x$ equal to each of these. We end up getting the following:

   $5x = \pi/2$, so $x = \pi/10$

   $5x = 3\pi/2$, so $x = 3\pi/10$

   $5x = 5\pi/2$, so $x = 5\pi/10$

   etc.

   From here we see the general pattern that the solutions will be odd multiples of $\pi/10$, namely $\pm\pi/10$, $\pm 3\pi/10$, $\pm 5\pi/10$, $\pm 7\pi/10$, etc. We could write this as $n\pi/10$, where n can be any odd integer, positive or negative.

2. We know that sine is 0 at all multiples of π. So we set $2x + 1$ equal to each of these. We end up with the following:

   $2x + 1 = 0$, so $x = -1/2$

   $2x + 1 = \pi$, so $x = (\pi - 1)/2$

   $2x + 1 = 2\pi$, so $x = (2\pi - 1)/2$

   $2x + 1 = 3\pi$, so $x = (3\pi - 1)/2$

   etc.

   The general pattern of solutions will be $x = (n\pi - 1)/2$ where n can be any integer.

**************** DRAFTS ****************