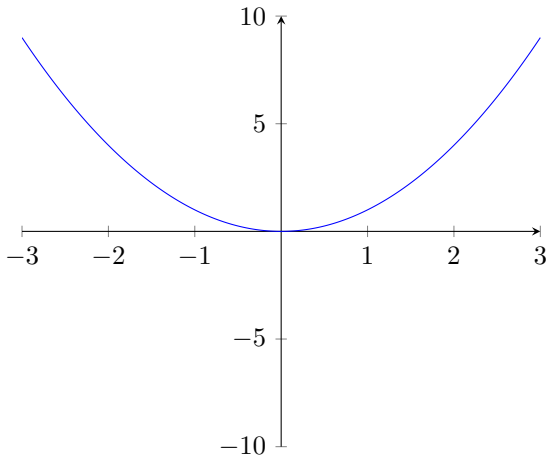
Graphs of functions

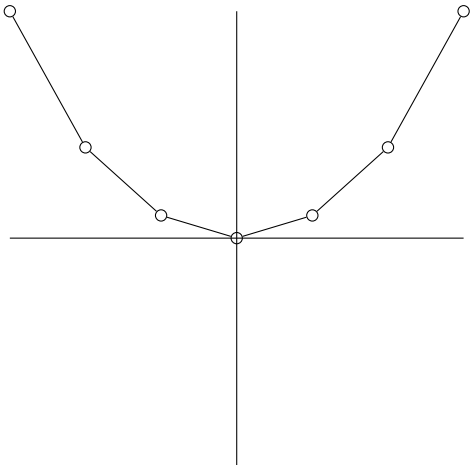
A graph is way of visualizing functions. For example, here is a graph of $f(x) = x^2$:



Plotting graphs

To plot a graph by hand, one approach is to make a table of values, plot each point, and connect them, like in the example below plotting $f(x) = x^2$.

x	$f(x)$
-3	9
-2	4
-1	1
0	0
1	1
2	4
3	9



The more points you can do, the better the plot will be. That's what makes computer graphing so nice—the computer can literally plot thousands of points.

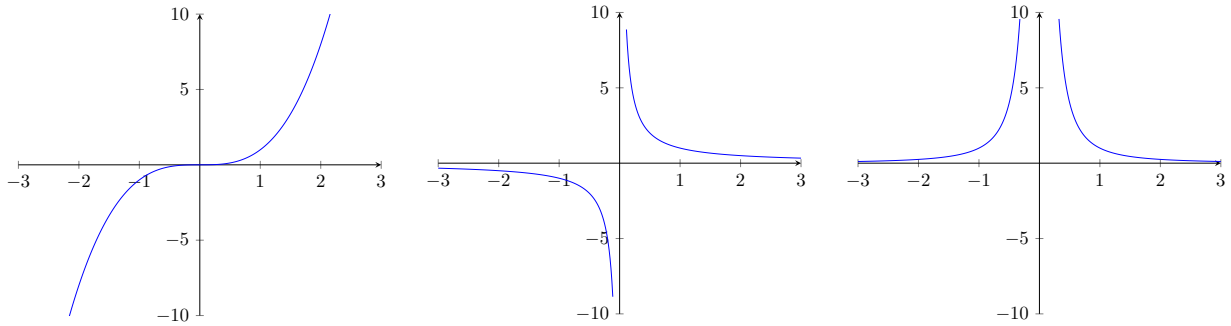
Useful graphs to know

There are some graphs that are very useful to know. In particular, know the graphs of

- Lines (linear equations, like $f(x) = mx + b$)
- The parabola $f(x) = x^2$
- The cubic equation $f(x) = x^3$.

- $f(x) = \dfrac{1}{x}$ and $f(x) = \dfrac{1}{x^2}$
- The trig functions $\sin x$, $\cos x$, and $\tan x$
- The exponential and logarithm functions e^x and $\ln x$.

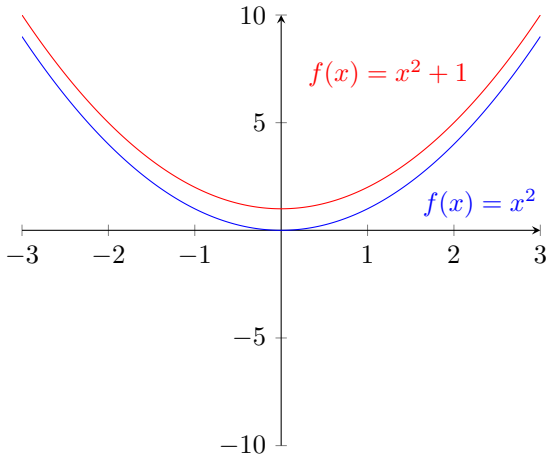
A few of these are shown below.



$f(x) = x^3$ $f(x) = \frac{1}{x}$ $f(x) = \frac{1}{x^2}$

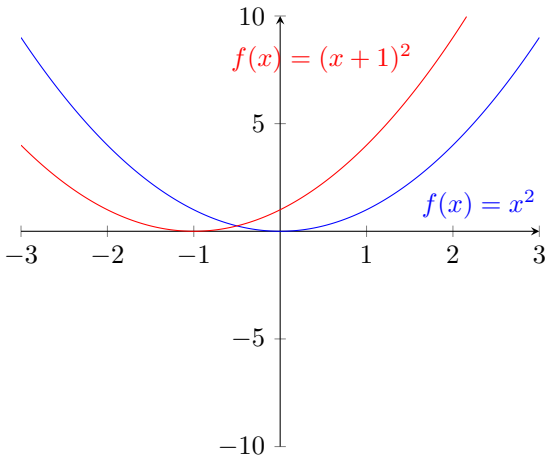
Using graphs we know to graph other functions

A useful plotting skill is to be able to use the graphs we know to get other ones. For example, we know that $f(x) = x^2$ is a parabola. It turns out that $f(x) = x^2 + 1$ is also a parabola, just shifted up 1 unit, like shown below.



This works in general: the graph of $f(x) + a$ will be the same as the graph of $f(x)$, just shifted vertically by a units (up if a is positive, down if a is negative).

While $x^2 + 1$ is a vertical shift, $(x + 1)^2$ is a horizontal shift, in this case by 1 unit left. See below.



In general, $f(x+a)$ is same shape as $f(x)$, just shifted by a units left or right, depending on if a is positive or negative.

Exercises

1. By plotting points, sketch the graph of $f(x) = \dfrac{1}{1+x^2}$.

2. Use the graphs given earlier in the section and the rules for horizontal and vertical shifts to graph the following.

   (a) $f(x) = x^2 - 2$

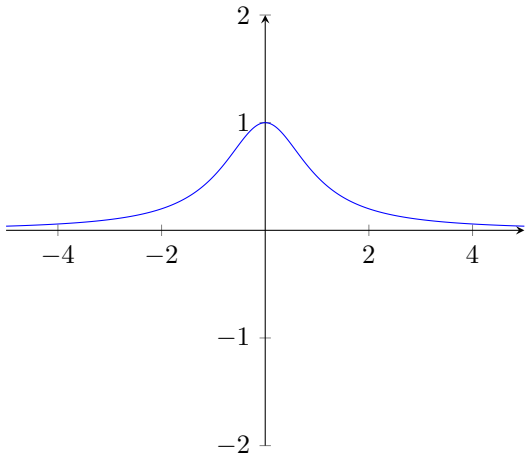
   (b) $f(x) = (x-2)^2$

   (c) $f(x) = (x-3)^2 + 4$

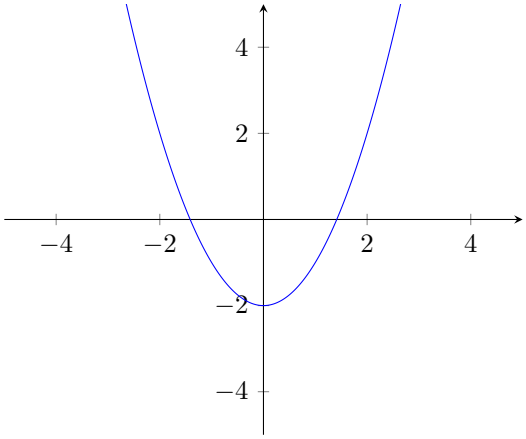
   (d) $f(x) = \dfrac{1}{x-1}$

Answers

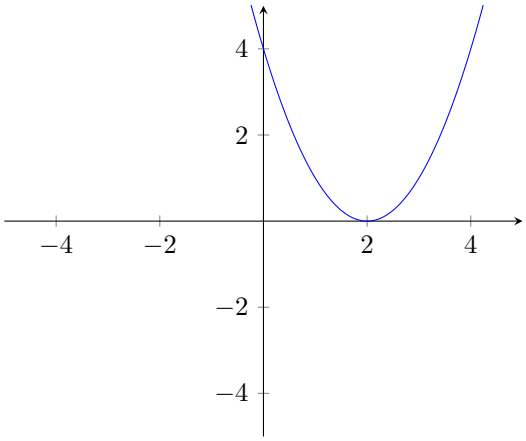
1. Pick several points, like maybe $x = -5, -4, -3, -2, -1, 0, 1, 2, 3, 4, 5$. The graph should look about like this:



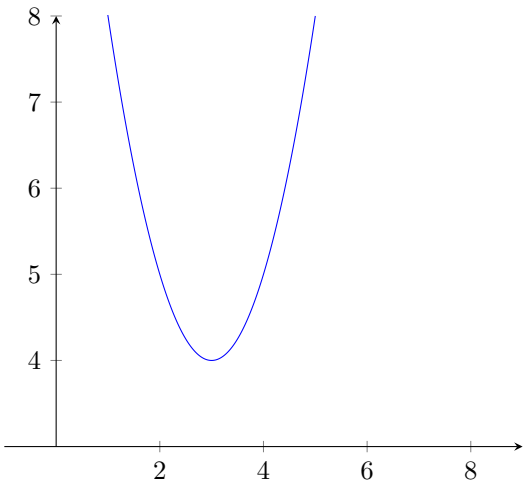
2. (a) This is the graph of the parabola x^2 shifted down by 2 units.



(b) This is the graph of x^2 shifted right 2 units.



(c) This is the graph of x^2 shifted right 3 units and up 4 units.



(d) This is the graph of $1/x$ shifted right 1 unit.