Factoring quadratics

Being able to factor quadratics (polynomials where the highest power is 2) is useful to know for calculus. There are a few common techniques to know.

Factoring expressions of the form $x^2 - y^2$

$x^2 - y^2$ can be factored into $(x - y)(x + y)$. This shows up all the time. Here are some examples:

1. $x^2 - 1 = (x - 1)(x + 1)$
2. $x^2 - 4 = (x - 2)(x + 2)$
3. $9 - x^2 = (3 - x)(3 + x)$
4. $4x^2 - 9 = (2x - 3)(2x + 3)$
5. $x^4 - 1 = (x^2 - 1)(x^2 + 1)$ (This isn't exactly a quadratic, but follows same idea.)

Note: The expression $x^2 + y^2$ cannot be factored.

Factoring expressions of the form $x^2 + bx + c$

Quadratics of the form $x^2 + bx + c$ can sometimes be factored into two terms. The approach uses two numbers that multiply together to equal c and at the same time add up to give b. Signs are important. Here are a few examples:

Example 1 Factor $x^2 + 7x + 12$.

Solution: We want two numbers that multiply together to give 12 and that add up to 7. After a little trial and error, it turns out that 3 and 4 work. The factorization is $(x + 3)(x + 4)$. You can check by multiplying this out that it is the same as $x^2 + 7x + 12$.

Example 2 Factor $x^2 - 9x + 8$.

Solution: We want two numbers that multiply together to give 8 and add up to -9. In this case, -4 and -2 work. The factorization is $(x - 4)(x - 2)$.

Example 3 Factor $x^2 + 3x - 10$.

Solution: We want two numbers that multiply together to give -10 and add up to give 3. If we use 5 and -2, that will work. The factorization is $(x + 5)(x - 2)$.

Note: It's not always possible to factor quadratics like this. For instance, $x^2 + 7x + 1$ can't be done.

Factoring expressions of the form $ax^2 + bx + c$

If the x^2 term has a coefficient that is not 1, a related form of this factoring works. Here are some examples:

Example 1 Factor $2x^2 + 13x + 15$.

Solution: The factorization that works is $(2x + 3)(x + 5)$. The way we get this is by doing the FOIL process in reverse. The two first terms of each factor have to multiply to give $2x^2$. The only way this will work is if one is $2x$ and the other is x. The two last terms have to multiply to give 15, and the cross terms have to add up to give $13x$. This makes it a bit like a puzzle where we have to work out what factors of 15 will work and in what places so that everything works out.

Example 2 Factor $6x^2 + 13x - 8$.

Solution: Think of it as a puzzle where have an expression like $(\_x + \_)(\_x + \_)$ and we have to fill in the blanks and choose signs so that the multiplication comes out to $6x^2 + 13x - 8$. To get the terms in front of the x terms, our possibilities are $2x$ and $3x$ or x and $6x$. For the other terms, they have to multiply together to give -8. And everything has to be chosen so that the cross term comes out to $13x$. After some trial and error and educated guessing, $(2x - 1)(3x + 8)$ turns out to work.

Exercises

Factor the following.

1. $x^2 - 16$
2. $25 - 9x^2$
3. $x^2 + 9x + 20$
4. $x^2 - 4x - 77$
5. $2x^2 - 14x - 36$
6. $8x^2 + 22x + 5$

Answers

1. $x^2 - 16 = (x - 4)(x + 4)$

2. $25 - 9x^2 = (5 - 3x)(5 + 3x)$

3. $x^2 + 9x + 20 = (x + 5)(x + 4)$

4. $x^2 - 4x - 77 = (x - 11)(x + 7)$

5. $2x^2 - 14x - 36 = (2x + 4)(x - 9)$

6. $8x^2 + 22x + 5 = (4x + 1)(2x + 5)$