Computing sines and cosines

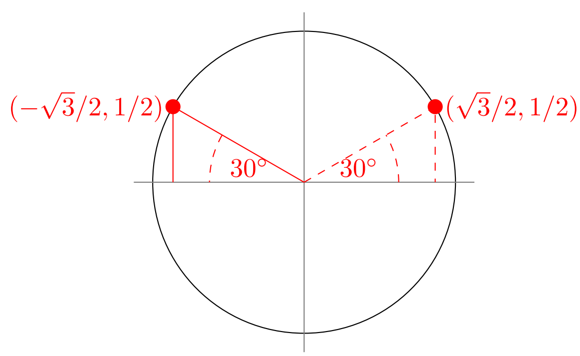
Using the unit circle and the 30-60-90 and 45-45-90 triangles gives us all the values of sine and cosine that students are typically expected to know. Here is a handy table for remembering some of these values.

angle	sine	cosine
0	$\sqrt{0}/2$	$\sqrt{4}/2$
$\pi/6$	$\sqrt{1}/2$	$\sqrt{3}/2$
$\pi/4$	$\sqrt{2}/2$	$\sqrt{2}/2$
$\pi/3$	$\sqrt{3}/2$	$\sqrt{1}/2$
$\pi/2$	$\sqrt{4}/2$	$\sqrt{0}/2$

To use this table, you would want to do a little simplification. For instance, $\sin(\pi/6)$ is listed as $\sqrt{1}/2$, but since $\sqrt{1}$ is 1, we see $\sqrt{1}/2$ is really 1/2 in disguise.

Angles greater than $90°$

You can combine the values in this table along with the unit circle to figure out sines and cosines for certain other angles. For example, suppose we want $\sin(5\pi/6)$. Personally, I find it easiest to convert this first to degrees. Multiply $5\pi/6$ by $180/\pi$ to get $150°$. This is shown on the unit circle below:



The $150°$ angle means the point on the unit circle is $30°$ above the horizontal but on the opposite side of $30°$. So since $30°$ corresponds to the point $(\sqrt{3}/2, 1/2)$, $150°$ will have the exact same height and the same x-coordinate except negative, namely $(-\sqrt{3}/2, 1/2)$. So $\cos(5\pi/6) = -\sqrt{3}/2$ and $\sin(5\pi/6) = 1/2$.

Zeroes of sine and cosine

It's also very handy to know when sine and cosine are 0. Here are the rules:

- Sine is 0 at integer multiples of π, namely 0, $\pm\pi$, $\pm 2\pi$, $\pm 3\pi$, etc.
- Cosine is 0 at odd integer multiples of $\pi/2$, namely $\pm\pi/2$, $\pm 3\pi/2$, $\pm 5\pi/2$, $\pm 7\pi/2$ etc.

Note that whenever one of sine or cosine is 0, the other will either be 1 or -1.

Calculators

Professors vary in terms of how much of this they want you to know by heart. Any scientific or graphing calculator will have buttons for sine and cosine. Just be careful to make sure you have it set for the proper mode (degree or radian). It's a good idea to keep it in radian mode.

Exercises

1. Without using a calculator, find the cosine and sine of the following angles.
    (a) $\pi/6$
    (b) $\pi/2$
    (c) 2π
    (d) 8π
    (e) $5\pi/2$
    (f) $2\pi/3$
    (g) $4\pi/3$
    (h) $11\pi/6$
    (i) $5\pi/4$
2. Use a calculator to estimate the cosine and sine of the following angles to 4 decimal places. (all are in radians).
    (a) $3\pi/5$
    (b) 1
    (c) .025

Answers

1. (a) $\cos(\pi/6) = \sqrt{3}/2$, $\sin(\pi/6) = 1/2$ — It's helpful to memorize sine and cosine of 0, 30, 45, 60, and 90 (i.e. 0, $\pi/6$, $\pi/4$, $\pi/3$, and $\pi/2$).

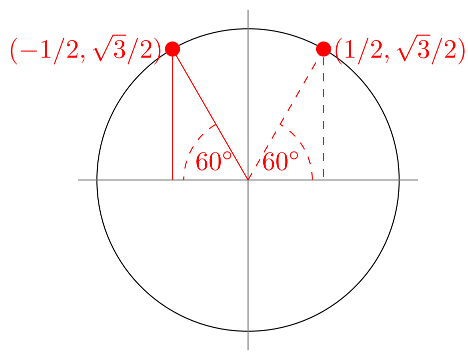
   (b) $\cos(\pi/2) = 0$, $\sin(\pi/2) = 1$ — On the unit circle, $\pi/2$ is the point $(0, 1)$ and cosine is the x-coordinate (0) and sine is the y-coordinate (1.

   (c) $\cos(2\pi) = 1$, $\sin(2\pi) = 0$ — 2π is in the same location on the unit circle as 0, namely the point $(1, 0)$.

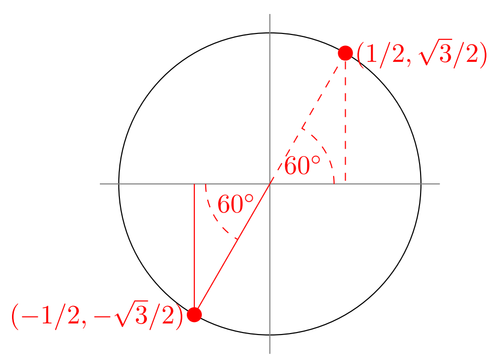
   (d) $\cos(2\pi) = 1$, $\sin(2\pi) = 0$ — 8π is in the same location on the unit circle as 0, namely the point $(1, 0)$.

   (e) $\cos(5\pi/2) = 0$, $\sin(5\pi/2) = 1$, — $5\pi/2$ (2π is in the same location on the unit circle as $\pi/2$, namely the point $(0, 1)$.)

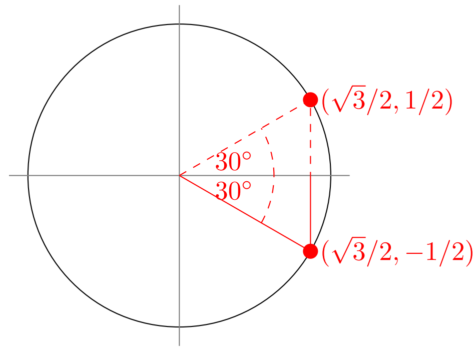
   (f) $\cos(2\pi/3) = -1/2$, $\sin(2\pi/3) = \sqrt{3}/2$ — First, $2\pi/3$ radians is 120°, shown below on the unit circle. We see it corresponds to an angle of 60° in the first quadrant. So we use what we know about sine and cosine of 60° and reflect across the axis to get the coordinates of 120°.



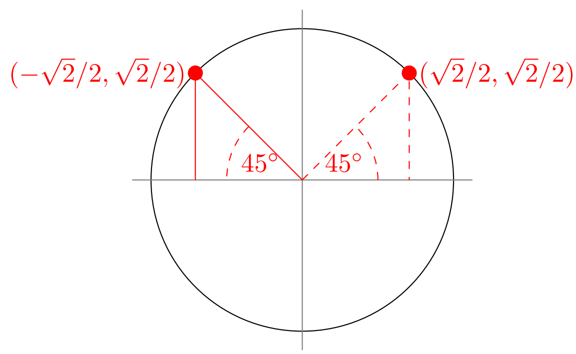
   (g) $\cos(4\pi/3) = -1/2$, $\sin(4\pi/3) == \sqrt{3}/2$ — First, $4\pi/3$ radians is 240°, shown below on the unit circle. We see it corresponds to an angle of 60° in the first quadrant. So we use what we know about sine and cosine of 60° and reflect across the axes to get the coordinates 240°.



   (h) $\cos(11\pi/6) = \sqrt{3}/2$, $\sin(11\pi/6) = -1/2$ — First, $11\pi/6$ radians is 330° or −30°, shown below on the unit circle. We see it corresponds to an angle of 30° in the first quadrant. So we use what we know about sine and cosine of 30° and reflect across the axis to get the coordinates of 330°.



(i) $\cos(5\pi/4) = -\sqrt{2}/2$, $\sin(5\pi/4) = \sqrt{2}/2$ — First, $5\pi/4$ radians is $135°$, shown below on the unit circle. We see it corresponds to an angle of $45°$ in the first quadrant. So we use what we know about sine and cosine of $45°$ and reflect across the axis to get the coordinates of $135°$.



2. (a) $\sin(3\pi/5) = 0.9511$, $\cos(3\pi/5) = -0.309$

   (b) $\sin(1) = 0.8415$, $\cos(1) = 0.5403$

   (c) $\sin(.025) = 0.0250$, $\cos(.025) = 0.9997$