200 Years/200 Questions — Week 1

You may answer as many questions as you like. Email solutions to contest@msmary.edu by Friday August 31.

1. Suppose you are at the fountain of youth. You must drink *exactly* 6 ounces of water from the fountain in order to obtain eternal youth. Just a bit too much or too little might have the opposite effect! The problem is, you only have two empty containers with you: a 7-ounce container, and an 11-ounce.

   You can take as much water as you like from the fountain, and you can pour water from container to container. However, you must get this exactly right! You can't guestimate that a container is roughly half-full. In other words, each time water is poured into a container, one of the containers must be completely filled or completely emptied. How can you get exactly 6 ounces?

2. Two perfectly cylindrical buckets, one with a diameter of 20 cm and the other one with a 30 cm diameter, are placed empty in the garden. During the night it rains, and in the morning they are collected. In the first bucket the rain is 8 cm deep. How high is the water in the second bucket?

3. Find the 1808th digit in the decimal expansion of $\frac{1}{13}$. Briefly explain how you got your answer.

4. What is the sum of all odd integers from 1 to 2008?

5. Professor Portier baked a cake for the Math Department meeting. Professor D. ate 1/6 of the cake. Professor Weiss ate 1/5 of what was left. Professor Heinold ate 1/4 of what was left after that. Professor Butler ate 1/3 of what was left after that. Professor Petrelli ate 1/2 of what was left after that. How much of the original cake was left for Professor Portier to eat? Who ate the most cake?

6-7. Below are pairs of words of the same length. Your challenge is to change the first word to the other word by forming other successive words (real words) of the same length. You are allowed to change only one letter at a time. Beside each pair is a number indicating the number of changes allowed. The first one is done for you.

   Four to Five (7)

   Answer: Four → Foul → Fool → Foot → Fort → Fore → Fire → Five

   6. Eye to Lid (3)
   7. Pity to Good (7)

The 200 Years/200 Questions contest is being offered by the Department of Mathematics and Computer Science at Mount Saint Mary's. Seven new problems will be posted each week until we reach 200 at the end of the spring semester. The contest is open to the entire Mount community. Visit http://faculty.msmary.edu/heinold/contest/ for details on *prizes*.

200 Years/200 Questions — Week 2

You may answer as many questions as you like. Email solutions to contest@msmary.edu by Friday September 7.

8. The currency of a (strange) country has coins worth 7 cents and 11 cents. What is the largest purchase price that cannot be paid using these two coins?

9. At a Mount St. Mary's Bicentennial event, students arrive in different groups. You are alone and the first to arrive at the event, but you are still considered a group. The second group to arrive has 2 more students in it than were in your group. The third group has 2 more students than were in the second group. If 625 students arrived at the Bicentennial Event on this day, how many different groups arrived at the school, assuming that they all met the requirement of having 2 more members than the group before them?

10. Professor Portier drinks a jug of wine in 15 days. If Professor Petrelli helps him, the jug will be empty in 6 days. How many days would it take Professor Petrelli to drink the jug all by himself?

11. A woman was out on her first bear hunt when she heard a noise. Nervously, she turned around to see a huge bear 100 yards due east of her. Panic stricken, she ran due north. Having covered about 100 yards, she regained her composure, stopped, and turned to shoot the bear. She was now facing due south and the bear had not moved.

    What color was the bear?

12. Given that $f(x) > 0$, solve for $f(17)$, where

    $$f(x) = \frac{x}{x + \frac{x}{x + \cdots}}$$

    Do not merely give a decimal approximation!

Below are pairs of words of the same length. Your challenge is to change the first word to the other word by forming other successive words (real words) of the same length. You are allowed to change only one letter at a time. Beside each pair is a number indicating the maximum number of changes allowed. The first one is done for you.

Four to Five (7)

Answer: Four → Foul → Fool → Foot → Fort → Fore → Fire → Five

13. Wood to Tree (8)

14. Oat to Rye (4)

The 200 Years/200 Questions contest is being offered by the Department of Mathematics and Computer Science at Mount Saint Mary's. Seven new problems will be posted each week until we reach 200 at the end of the spring semester. The contest is open to the entire Mount community. Visit http://faculty.msmary.edu/heinold/contest/ for details on prizes.

200 Years/200 Questions — Week 3

Answer as many questions as you like. Email solutions to contest@msmary.edu by Friday September 14.

15. You pull out a page from a newspaper and find that pages 8 and 21 are on the same sheet. How many pages does the newspaper have?

16. One of the numbers in the list below is not like the others. Can you spot it? (Please give a reason for your choice.)

    $$\{11, 17, 19, 23, 41, 51, 53, 67, 83, 89\}$$

17. Before coming to Mount Saint Mary's, Professor Heinold was marooned on a strange island inhabited by trolls. The trolls belonged to two tribes. One tribe's members always told the truth, while the other tribe's members always lied. One day Professor Heinold met three trolls. He asked them which tribes they belonged to.

    "All of us are Liars," said the first.

    "No, only two of us are Liars," amended the second.

    "That's not true either," corrected the third. "Only one of us is a Liar."

    To which tribe did each belong?

18. Baseball — Believe it or not, it is possible for player A to have a higher batting average than player B in both day games *and* in night games, even though Player B has a higher average overall. Give an example showing how this could happen.

19. What fraction becomes .18081808 . . . (1808 repeated forever) when written as a decimal?

20. The product of the ages of a group of teenagers is 10,584,000. Find the number of teenagers and their ages.

21. For each of the words and phrases below, drop one letter and rearrange the rest to get a single word that is associated with a button on a calculator. For example, if the word were PULSE, you'd drop the E and rearrange the rest to get PLUS.

    (a) NIMBUS
    (b) OVID, DIE!
    (c) RECOINS
    (d) SPICE CORRAL

The 200 Years/200 Questions contest is being offered by the Department of Mathematics and Computer Science at Mount Saint Mary's. Seven new problems will be posted each week until we reach 200 at the end of the spring semester. The contest is open to the entire Mount community. Visit http://faculty.msmary.edu/heinold/contest/ for details on prizes.

200 Years/200 Questions — Week 4

Answer as many questions as you like. Email solutions to contest@msmary.edu by Friday September 21.

22. What letter comes next in the sequence below?

    n, w, h, o, i, i, e, ?

23. The U.S. open tennis tournament just concluded on Sunday. Roger Federer won for the fourth year in a row, and Justine Henin won for the second time. In the men's draw there were 128 players. It is a single-elimination tournament, which means that if a player loses one match, then he's out of the tournament. How many men's matches must have been played in total? (Assume every possible match was played.)

24. Suppose we group the odd numbers as below. What is the sum of the numbers in the 100th group?

    $1; \quad 3, 5; \quad 7, 9, 11; \quad 13, 15, 17, 19;$ *etc.*

25. How many perfect squares are there between 2008 and 8008?

26. What is the minimum number of people needed in a group so that you would be certain that at least 3 of them were born in the same month and on the same day of the week?

27. Professor Jarvis lives on a large hill. The number of stairs leading up to her house is such that whether you climb the stairs two at a time, three at a time, four at a time, or even five at a time, you will always wind up one stair short of the top. There are less than 100 stairs leading up to Professor Jarvis' house. How many stairs are there?

28. Five professors from Math and Computer Science and their five dogs were having a lot of fun camping. Each professor owned one of the dogs. While they were all on a hike, they came to a river they had to cross. There was a small motorboat alongside the river. They determined that the boat was only large enough to hold three living things—either dogs or professors. Unfortunately, the dogs were a little temperamental, and they really didn't like the professors. Each dog was comfortable with its owner and no other professor. Each dog could not be left with the professors unless the dog's owner was present—not even momentarily! Dogs could be left with other dogs, however. The crossing would have been impossible except Professor Petrelli's dog, which was very smart, knew how to operate the boat. None of the other dogs were that smart, however. How was the crossing arranged, and how many trips did it take?

The 200 Years/200 Questions contest is being offered by the Department of Mathematics and Computer Science at Mount Saint Mary's. Seven new problems will be posted each week until we reach 200 at the end of the spring semester. The contest is open to the entire Mount community. Visit http://faculty.msmary.edu/heinold/contest/ for details on prizes.

200 Years/200 Questions — Week 5

Answer as many questions as you like. Email solutions to contest@msmary.edu by Friday September 28.

29. Sometimes it is possible to answer a question without having all of the information. Suppose you are told that a softball player has a batting average of .309, and you are asked: Does the player have 15 hits, 25 hits, or 28 hits? How can you answer this question without knowing how many at bats the player has?

30. After a typist had written ten letters and had addressed the ten corresponding envelopes, a careless mailing clerk inserted the letters in the envelopes at random, one letter per envelope. What is the probability that exactly nine letters were put in the proper envelopes?

31. Those who have taken a class with Professor Heinold know that he often makes arithmetic mistakes. To keep his Ph.D. from being taken away, last week he had to take a 20-question arithmetic exam that was scored in this way: 10 points were awarded for each correct answer and 5 points were deducted for each incorrect answer. He answered all 20 questions and received a score of 125 points. How many questions did he get wrong?

32. Find the error in the following calculation:

$$1 = \sqrt{1} = \sqrt{(-1)(-1)} = \sqrt{-1}\sqrt{-1} = i \cdot i = -1.$$

33. Consider a list of numbers consisting of one 1, two 2s, three 3s, and so on until we reach nine 9s, after which the sequence repeats. (So the first few terms of the sequence are: 1, 2, 2, 3, 3, 3, 4, 4, 4, 4, 5, . . .) What is the 1808th term in this sequence?

34. Using four 4s, the symbols $+$, $-$, $\times$ and $\div$, and parentheses, it is possible to get any number between 1 and 9. For instance, you can get 1 by doing $(4+4) \div (4+4)$. Another way is $(4+4-4) \div 4$. You could even do $44 \div 44$. Can you show how to get the numbers 2 through 5?

35. Factorials — $n!$ is the product of all the integers between 1 and n. For example, $3! = 3 \times 2 \times 1 = 6$, $4! = 4 \times 3 \times 2 \times 1 = 24$, and $5! = 5 \times 4 \times 3 \times 2 \times 1 = 120$. Factorials get very large very fast. In fact, even 100! is too much for a typical graphing calculator to handle! However, you can do the following problem without a calculator: How many zeros does 1000! end with?

The 200 Years/200 Questions contest is being offered by the Department of Mathematics and Computer Science at Mount Saint Mary's. Seven new problems will be posted each week until we reach 200 at the end of the spring semester. The contest is open to the entire Mount community. Visit http://faculty.msmary.edu/heinold/contest/ for details on prizes.

200 Years/200 Questions — Week 6

Answer as many questions as you like. Email solutions to contest@msmary.edu by Friday October 5.

36. If to the numerator and denominator of the fraction $\frac{1}{3}$, you add its denominator, 3, the fraction will double: $\frac{1+3}{3+3} = \frac{2}{3}$. Can you find a fraction such that when you add its denominator to both the numerator and denominator, the new fraction becomes 1808 times the original fraction?

37. What are the next two numbers in the series 2, 10, 4, 7, 8, 4, ...?

38. Six knights are sitting at the round table, and two are selected at random. What are the chances that they are sitting next to each other?

39. Last week we asked you to show how to get the numbers 2 through 5 using four 4s, the symbols $+$, $-$, $\times$, $\div$, and parentheses. The example we gave was that you can get 1 by doing $(4+4) \div (4+4)$. Another way is $(4+4-4) \div 4$. You could even do $44 \div 44$. To get the numbers 6 through 10 is slightly harder than 2 through 5, but it can still be done. Can you show how?

40. In algebra class you were probably asked lots of times to find the roots of a polynomial. For instance, the roots of $x^2 - 5x + 6$ are $x = 2$ and $x = 3$. Have you ever tried to go backwards? Try this: What polynomial has $\sqrt{3+\sqrt{2}}$ as one of its roots?

41. The fifth floor of the Science Building is rather strange. It has an infinite number of rooms. Every *natural* number is a possible room number, *e.g.* 1, 2, 3, 4, etc. One day, professor Portier decided to play around. Initially, the door to every room was closed. He decided to open every door. After opening every door, he then decided to close every second door, starting with the second door.

    Then he decided to open or close every third door, starting with the third door, according to the rule: if the door was open, he closed it, and if the door was closed, he opened it. He then used the same rule for every fourth door, starting with the fourth door, and, in fact, he used the same rule for every natural number n starting with the nth door. (Professor Portier must not have been busy that day!)

    When he was finished, which doors were open and which were closed? Why?

42. Here's a problem that is often asked of people on job interviews.

    > There are four people who need to cross a rickety old bridge at night. It is too dark to see, so they need a flashlight to get across, and the bridge can only hold two of them at a time. Each can cross the bridge at a different rate: the times are 10 minutes, 5 minutes, 2 minutes, and 1 minute for each of the four people. It is very dark, and someone must bring the flashlight back to the others. How fast can they all get across? (There can be no halfway crosses or throwing the flashlight across the bridge, or any other wily tricks, and the answer is strictly less than 19 minutes!)

The 200 Years/200 Questions contest is being offered by the Department of Mathematics and Computer Science at Mount Saint Mary's. Seven new problems will be posted each week until we reach 200 at the end of the spring semester. The contest is open to the entire Mount community. Visit http://faculty.msmary.edu/heinold/contest/ for details on prizes.

200 Years/200 Questions — Week 7

Answer as many questions as you like. Send solutions to contest@msmary.edu by Friday October 19.

43. The letters in the phrase MAIDEN NAME can be arranged to spell two related mathematical terms. What are they?

44. Fifteen professors are about to sit around a table. Before sitting they shake hands with each other. If every professor shakes hands with every other professor, how many handshakes have there been?

45. Can you find a number which, when multiplied by 1808, equals 18081808? How about a number which, when multiplied by 1808, equals 180818081808? Can you explain what to multiply 1808 by to get $18081808 \cdots 1808$ in general?

46. Jeff and Bubba robbed a grocery store and went around bragging about it. There is a 50% chance that Bubba will get caught because of his bragging, and there is a 50% chance that Jeff will get caught because of his bragging. If either gets caught, there is a 50% chance he will betray the other one. What is the chance that precisely one gets caught and not the other?

47. In week 2 of the contest we had a question about a woman out on a bear hunt. Let's restate that problem slightly. Suppose the woman walks one mile due south, turns and walks one mile due east, turns again and walks one mile due north. She finds herself back exactly where she started, and then shoots a bear. What color is the bear?

    You may remember that the answer is *white*, because she started at the North Pole. (It's a polar bear.) But not long ago someone made the discovery that the North Pole is not the only starting point that satisfies the given conditions! Can you think of any other spot on the globe from which the woman could walk a mile south, a mile east, a mile north and find herself back at her original location?

48. Two logicians are admiring a garden containing at least one pink, one yellow, and one blue flower. The first logician remarked correctly that if you pick any three flowers in the garden, at least one must be pink. The second remarked that if you pick any three flowers, at least one must be yellow. How many flowers are in the garden? Briefly explain your answer.

49. Two professors, one of English and one of mathematics, were having drinks in the faculty club bar.

    "It is curious," said the English professor, "how some poets can write one immortal line and nothing else of lasting value. John William Burgon, for example. His poems are so mediocre that no one reads them now, yet he wrote one of the most marvelous lines in English poetry: 'A rose-red city half as old as Time.' "

    The mathematician, who liked to annoy his friends with improvised brainteasers, thought for a moment or two, then raised his glass and recited:

    A rose-red city half as old as Time
    One billion years ago the city's age
    Was just two-fifths of what Time's age will be
    A billion years from now. Can you compute
    How old the crimson city is today?

    Do you remember enough algebra to find the answer? ("No" is not an acceptable solution. Neither is "Yes," for that matter.)

200 Years/200 Questions — Week 8

Answer as many questions as you like. Send solutions to contest@msmary.edu by Friday October 26.

50. I am thinking of a number that is a repeating decimal of the form 0.abcdabcd.... Find my number using the following clues.

    (a) The digit that is located 100 places after the decimal is 4.
    (b) The digit that is 150 places after the decimal is 5.
    (c) The digit that is 175 places after the decimal is 3.
    (d) $a+b=c+d$

51. During a period of days, it was observed that when it rained during the afternoon, it had been clear in the morning, and when it rained in the morning, it was clear in the afternoon. It rained on 9 days and was clear on 6 afternoons and 7 mornings. How long was the period?

52. Before becoming a math professor, Professor Portier was a milkman. Two women once asked him for 2 quarts of milk apiece. One had a 5-quart pail, and the other had a 4-quart pail. Professor Portier had only two 10-gallon cans, each full of milk. How did he measure out exactly 2 quarts of milk for each woman? (He did it without any tricks, using only the two cans and two pails, and did no estimating.)

53. Professor Weiss purchased some sugar by the pound and paid a total of $2.16. Had the sugar cost 1 cent a pound less, he would have received 3 pounds more for the same expenditure. How many pounds of sugar did Professor Weiss buy?

54. A peasant wanted to marry the mayor's daughter. The mayor summoned the peasant and all the villagers to the town square. The ground of the square was littered with a mix of thousands of black and white pebbles. The mayor announced to everyone, "Everyone knows I'm a fair man, so I will give this suitor a chance. I'll draw a white pebble and a black pebble from the ground and place one randomly in each of my fists. The suitor will choose one hand. If it contains a white pebble, he can marry my daughter. If it is a black pebble, he must leave our village and never return." The mayor reached down to the stones. The peasant, watching closely, was horrified to see that the mayor surreptitiously picked up two black stones! He was sure that if he accused the mayor of cheating, he would be thrown out of the village and lose his chance to marry the daughter. Fortunately, he was a bright man, and he came up with a way to go through the contest without accusing the mayor and still get to marry the daughter. What did he do?

55. What does the following product equal?

$$\left(1-\frac{1}{2^2}\right)\times\left(1-\frac{1}{3^2}\right)\times\left(1-\frac{1}{4^2}\right)\times\cdots\times\left(1-\frac{1}{1808^2}\right)$$

56. Two boats on opposite shores of a lake leave at the same time. Each travels at a constant speed, though one travels faster than the other. When they first meet, the nearest shore to the boats is 700 meters away. Each then continues to the opposite shore, and each spends 10 minutes on the shore before setting off again. This time, when they meet, they are 400 meters from the other shore. How wide is the lake?

The 200 Years/200 Questions contest is being offered by the Department of Mathematics and Computer Science at Mount Saint Mary's. Seven new problems will be posted each week until we reach 200 at the end of the spring semester. The contest is open to the entire Mount community. Visit http://faculty.msmary.edu/heinold/contest/ for details on prizes.

200 Years/200 Questions — Week 9

Answer as many questions as you like. Send solutions to contest@msmary.edu by Friday November 2.

57. A chicken and a half can lay an egg and a half in a day and a half. How many eggs can a chicken lay in six days?

58. A few weeks back, we had a problem where you were asked to use exactly four 4s and the symbols $+$, $-$, $\times$, and $\div$ to make the numbers 1 through 10. For this problem, in addition to the arithmetic symbols, you may also use the decimal point (.), the square root sign, ($\sqrt{\ }$), the factorial symbol (!), and the overbar that indicates a repeating decimal (for instance, $.\overline{4} = .44444\cdots$). Using these symbols, it is possible to get all of the numbers from 1 to 100, and, in fact, quite a bit further. Can you show how to get 13, 19, and 33?

59. What perfect square is the product of 4 consecutive odd integers?

60. Secure transfer of information over the internet is based on the fact that it's hard to factor large numbers. The number 999919 is actually the product of two fairly large prime numbers. Find them.

61. Find the sum of the *digits* appearing in the set of integers, $\{1, 2, 3, \ldots, 1808\}$. (For example, the sum of the digits appearing in the set $\{16, 17, 18\}$ is $1 + 6 + 1 + 7 + 1 + 8$, which equals 24.)

62. This one is a true story. Prof. Weiss was at a conference. For a potluck picnic, he brought some loaves of homemade banana bread in pans 12" long by 3" wide by 2" deep. Sadly, no one had the bread, so he took it back to his hotel room. The next morning, he wanted banana bread for breakfast, but he had no utensils to use to cut it. Fortunately, he had an idea, and without needing to leave his room or get assistance from anyone else, in seconds, he had perfectly sliced banana bread. What did he do? (We'll accept any plausible way to perfectly slice the bread.)

63. Let's suppose that you've had 2 calculus tests so far, and there are three classmates you can select for a study team for the third test, but you are allowed to select only one of them.

    The first classmate has an A on one of the tests and an F on the other. You don't know which test is the A test and which is the F test, so you may assume that there is a 50% chance that the student got an A on the first test. The second classmate has an F on both tests, and the third classmate has an A on both tests.

    You now select one of the three classmates at random, and then learn that he received an A on the first test. What is the chance that he received an A on both tests?

The 200 Years/200 Questions contest is being offered by the Department of Mathematics and Computer Science at Mount Saint Mary's. Seven new problems will be posted each week until we reach 200 at the end of the spring semester. The contest is open to the entire Mount community. Visit http://faculty.msmary.edu/heinold/contest/ for details on prizes.

200 Years/200 Questions — Week 10

Answer as many questions as you like. Send solutions to contest@msmary.edu by Friday November 9.

64. Here is a timely question, in more ways than one! Not too long ago, while Professor Dujmovic was visiting a certain east coast state, he called Professor August, who, after retiring from the Mount, now lives in a certain west coast state. They were both quite surprised to discover that it was the exact same time in both locations! How was this possible? (An east coast state is one that borders the Atlantic Ocean, and a west cost state is one that borders the Pacific. By the way, Alaska and Hawaii have nothing to do with the correct answer.)

65. What's the largest amount of money you can have in coins and still not be able to give change for a dollar?

66. A statistics student was working on an assignment, but got distracted by her roommate. When she went back to the problem, she had forgotten whether she was supposed to add or multiply the three different integers on her paper. She decided to do it both ways, and much to her surprise, the answer was the same. What were the three different integers?

67. Can you find 3 positive integers, a, b, and c that satisfy the equation $a^3 + b^3 + c^3 = (a + b + c)^2$? Can you find 1808 positive integers that satisfy an analogous equation? (1808 terms on each side)

68. How many telephone numbers are palindromes both with and without the area code included? A *palindrome*, in this case, means a number that reads the same backwards or forwards, like 2578752. Assume that any sequence of seven or ten digits constitutes a valid phone number.

69. How many of the numbers in the following infinite sequence are prime numbers? Please explain.

    9; 98, 987; 9876; 98765; . . . ; 987654321; 9876543219; 98765432198; 987654321987; *etc.*

70. It is said that Immanuel Kant was a bachelor of such regular habits that the good people of Königsberg would adjust their clocks when they saw him stroll past certain landmarks.

    One evening Kant was dismayed to discover that his clock had run down. Evidently his manservant, who had taken the day off, had forgotten to wind it. The great philosopher did not reset the hands because his watch was being repaired and he had no way of knowing the correct time. He walked to the home of his friend Schmidt, a merchant who lived a mile or so away, glancing at the clock in Schmidt's hallway as he entered the house.

    After visiting Schmidt for several hours Kant left and walked home along the route by which he came. As always, he walked with a slow, steady gait that had not varied in twenty years. He had no notion of how long this return trip took. (Schmidt had recently moved into the area and Kant had not yet timed himself on this walk.) Nevertheless, when Kant entered his house, he immediately set his clock correctly. How did Kant know the correct time?

The 200 Years/200 Questions contest is being offered by the Department of Mathematics and Computer Science at Mount Saint Mary's. Seven new problems will be posted each week until we reach 200 at the end of the spring semester. The contest is open to the entire Mount community. Visit http://faculty.msmary.edu/heinold/contest/ for details on prizes.

200 Years/200 Questions — Week 11

Answer as many questions as you like. Send solutions to contest@msmary.edu by Friday November 16.

71. What do the following have in common: The Greenwich Meridian, a fine roast rib of beef, television time from 7 to 10 P.M., and a positive integer n which divides the number $(n-1)!+1$?

72. Professor Heinold has taken to memorizing encyclopedias. He thinks they will make him smarter. Unfortunately, they haven't taught him any common sense. He stores the 12-volume set of encyclopedias in the following order from left to right: Volumes 8, 11, 5, 4, 9, 1, 7, 6, 10, 3, 12, and 2. Suppose the encyclopedia company comes out with a new volume, number 13. Between which two volumes will Professor Heinold place the new volume to be consistent with his system?

73. An *irrational number* is a number that cannot be expressed as a ratio of integers. So, while, for example, 2, 7/9 and $-22/7$ are *rational* numbers, $\sqrt{2}$, $\sqrt[3]{7}$, and π are irrational. Show, with a simple example, that an irrational number raised to an irrational power need not be irrational.

74. Construct an expression for as large a number as you can using only the digits 1, 2, 3, 4, parentheses, the decimal point, and the minus sign. Each digit may only be used once, but the symbols may be used multiple times. One example is $(4+3)^{2.1}$, which is roughly equal to 59.5. We will count any reasonable attempt as correct.

75. Sometimes problems are much easier than they appear. Evaluate the following radical:

$$\left(\frac{1\cdot 2\cdot 4\ +\ 2\cdot 4\cdot 8\ +\ 3\cdot 6\cdot 12\ +\ \cdots}{1\cdot 3\cdot 9 + 2\cdot 6\cdot 18\ +\ 3\cdot 9\cdot 27\ +\ \cdots}\right)^{1/3}$$

76. As we mentioned in week six's contest, the fifth floor of the Science Building is rather strange. There's one hallway the professors keep at a chilly $-180°$ centigrade. They don't want anyone messing with the thermostat, so they've rigged up a contraption that only a scientist would be able to figure out. It contains exactly 1808 switches, all initially on. In order to turn off the cold air, the switches must all be off. If even a single switch is on, the hallway will remain at $-180°$. The *only way* to flip the switches is a device which flips exactly 1807 of switches at a time, no more, no less. Describe a series of moves using this device that will turn all of the switches off.

77. Standard American roulette wheels have 38 numbers. A simple way to play the game is as follows: Let's say you bet \$1 that your favorite number, 7, is going to come out. The wheel is spun and a ball goes around until it randomly lands on one of the 38 numbers. If it lands on 7, you win \$35. Otherwise, you lose your money. Because the house only pays you \$35, and there are 38 numbers, the odds are stacked slightly in the casino's favor.

    However, here is a winning strategy for roulette. (Or is it?) Let's say you have \$50 to play with. You first play 25 times in a row, betting \$1 each time. If you manage to win at least once in those 25 tries, then you stop playing and walk away a winner. If you don't win in any of those 25 tries, you then play another 25 times, again betting \$1 each time. After those 25 tries, you stop playing, no matter what. What is the probability that you will end up with more money than you started with?

The 200 Years/200 Questions contest is being offered by the Department of Mathematics and Computer Science at Mount Saint Mary's. Seven new problems will be posted each week until we reach 200 at the end of the spring semester. The contest is open to the entire Mount community. Visit http://faculty.msmary.edu/heinold/contest/ for details on prizes.

200 Years/200 Questions — Week 12

Answer as many questions as you like. Send solutions to contest@msmary.edu by Friday November 23.

78. Something extraordinarily unusual happened on the 6th of May 1978, at thirty-four minutes past noon. What was it?

79. Professor Butler rode her bike 300 miles last summer. She used three tires equally in accumulating this distance. How many kilometers of wear did each tire sustain?

80. Two recent graduates, Ann and Bob, started working two years ago for different firms at the same salary. Last year Ann had a raise of 10% and Bob had a drop in pay of 10%. This year Ann had the 10% drop and Bob the 10% raise. Who's making more now? Please explain.

81. The number 7 can be expressed as a difference of two squares since $7 = 16 - 9$, which is $4^2 - 3^2$. In fact, any odd number and many even numbers can be expressed as a difference of two squares. Show how to express 1808 as a difference of two squares.

82. Describe how to express an arbitrary odd integer, $2n + 1$, as a difference of two squares.

83. Below is a *number spiral.* What are the next three numbers to the right of 19?

13	14	15	16	17			
12	3	4	5	18			
11	2	1	6	19	?	?	?
10	9	8	7	20			
		$\cdots$	22	21			

84. Find a formula to describe all of the numbers to the right of 19 in that row.

The 200 Years/200 Questions contest is being offered by the Department of Mathematics and Computer Science at Mount Saint Mary's. Seven new problems will be posted each week until we reach 200 at the end of the spring semester. The contest is open to the entire Mount community. Visit http://faculty.msmary.edu/heinold/contest/ for details on prizes.

200 Years/200 Questions — Week 13

Answer as many questions as you like. Send solutions to contest@msmary.edu by Monday December 3.

85. The 22nd and 24th presidents of the United States had the same mother and the same father, but were not brothers. How could this be possible?

86. Professor Heinold had a mean logic teacher in college. For the final exam the teacher told Professor Heinold to make one statement. If the statement was false, Professor Heinold would get an F for the course. If the statement were true, he would get a D. In order to graduate, he needed to do better than a D in the course. He thought about it for a minute, made a certain statement, and the teacher had no choice but to give him a passing grade (better than a D). What is the one thing Professor Heinold could have said?

87. The surface area and volume of a certain sphere are both four-digit integers times π. What is the radius of the sphere?

88. Professor Petrelli sold two classic calculators for \$600 each. On one he had a 20% profit, while on the other he had a 20% loss. Did he make money or lose money? How much?

89. Compute the sum $1 - 2 + 3 - 4 + \ldots + 1807 - 1808$, where the sign alternates with each term.

90. Here's one that's a little harder. Compute the sum $1^2 - 2^2 + 3^2 - 4^2 + \ldots + 1807^2 - 1808^2$.

91. A little algebra: Though it looks messsy, the expression $\left(\sqrt[6]{x^3} - \sqrt{2x + x/4}\right)^2$ can be simplified into something very simple. What is it? (Assume $x > 0$.)

The 200 Years/200 Questions contest is being offered by the Department of Mathematics and Computer Science at Mount Saint Mary's. Seven new problems will be posted each week until we reach 200 at the end of the spring semester. The contest is open to the entire Mount community. Visit http://faculty.msmary.edu/heinold/contest/ for details on prizes.

200 Years/200 Questions — Week 14

Answer as many questions as you like. Send solutions to contest@msmary.edu by Monday December 10.

92. More presidential trivia: John F. Kennedy was the youngest man ever elected president of the United States, yet he wasn't the youngest person ever to hold that office. How was that possible?

93. We've all heard the old saying that that even a stopped clock is correct twice a day. However, most clocks are not correct even once a day since they are most likely always a few seconds behind or ahead of the correct time!

    Professor Jarvis modified one of her clocks so that it was running at the normal speed and gave the correct time at least twice a day, even though she was not able to set it to the exact, correct time. What did she do?

94. A bricklayer has eight bricks. Seven of the bricks weigh the same, but one of them is a bit heavier than the others. How can the bricklayer use a balance scale to find the heaviest brick using only *two* weighings?

95. At 12 noon, the minute and hour hands of a clock are perfectly lined up. They next line up a little after 1:05pm. Precisely how many seconds after 1:05pm will it be?

96. Find the missing number: 3, 4, 7, 11, 18, 29, __, 76, 123.

97. It can get difficult to find primes once the numbers start getting large. Did you ever think it might be difficult to find non-primes? Find 1808 *consecutive* positive numbers, all of which are not prime.

98. Professor Weiss says his son's favorite game is *Go Away Monster*. There's a board representing a room, and a bag of tiles containing 16 pieces of furniture and 8 monsters. It's a cooperative game; on each turn, you draw a tile randomly from the bag. If it's a piece of furniture, you add it to the room. If it's a monster, you shout "Go Away Monster" and throw the tile away. (So once a tile is drawn, it doesn't go back in the bag.) The goal is to get all the pieces of furniture before you draw all the monsters. What is the probability of winning this game?

The 200 Years/200 Questions contest is being offered by the Department of Mathematics and Computer Science at Mount Saint Mary's. Seven new problems will be posted each week until we reach 200 at the end of the spring semester. The contest is open to the entire Mount community. Visit http://faculty.msmary.edu/heinold/contest/ for details on prizes.

200 Years/200 Questions — Week 15

Answer as many questions as you like. Send solutions to contest@msmary.edu by Friday January 25.

99. Can you find a plural, masculine noun, which, when you add exactly one letter to it, becomes a singular, feminine noun? (A masculine noun is one which is typically used to describe a male. “Boy” is one such word.)

100. Professor Petrelli did a lot of traveling over winter break. He rented a car in Thurmont and decided to see if it was possible, starting from the Mount, to drive through each of the contiguous 48 states and visit each state *exactly once*. He decided that he absolutely could not pass back through any state that he had previously visited during his trip. When he finally arrived in the last state, he dropped off his rental car and flew back to Maryland. What was the last state he visited?

101. Using each of the digits 0, 1, 2, . . . , 9 exactly once, find two five-digit numbers whose product is as large as possible.

102. What eight-letter term from geometry is an anagram of an eight-letter term from calculus?

103. Pens cost \$3 and pencils \$5. What is the greatest total number of pens and pencils that can be bought for exactly \$1808?

104. A teenager wants to go out on two consecutive nights of a three-day weekend. Each night, she has to ask either her father or mother for permission, but if the first one she asks says no, there’s no appealing to the other one. However, she knows that her father is more of a pushover than her mother. Moreover, if the same parent is asked two nights in a row, that parent will always give an answer on the second night that’s different from the previous night’s answer. Whom should she ask first in order to have the best chance of going out on two consecutive nights? Please explain your answer.

105. Professor Portier has a game to sell. He has four identical cups. Under cups there are numbers 3, 5, 9, and 0. On the outside it is not obvious what these numbers under the cups are. The player can start by selecting any cup. If the player selects 0, he gets nothing and the game is over. If he selects 3, 5, or 9 then he gets \$3, \$5, or \$9, according to what the selected cup was. Then Professor Portier shuffles the cups in a way that nobody can figure out where the numbers are again, even though a lot of people might think that they know. The same player can go on and on until he finally selects 0. At that point, the game is over and he walks away with whatever he’d won up to that point. How much money should Professor Portier charge per game to break even in the long run, not calculating any other costs (like cups, time etc)?

The 200 Years/200 Questions contest is being offered by the Department of Mathematics and Computer Science at Mount Saint Mary’s. Seven new problems will be posted each week until we reach 200 at the end of the spring semester. The contest is open to the entire Mount community. Visit http://faculty.msmary.edu/heinold/contest/ for details on prizes.

200 Years/200 Questions — Week 16

Answer as many questions as you like. Send solutions to contest@msmary.edu by Friday February 1.

106. As you probably are aware, 2008 is a leap year. Thursday, February 28th will be followed by Friday, February 29th. However, there was once a strange leap made in September. Wednesday September 2nd was immediately followed by Thursday September 14th, and all the intervening days were skipped over. Why?

107. Professor Butler was performing a delicate mathematical experiment that called for the room to be chilled to the precise temperature of -40 degrees. Just a fraction of a degree off and the experiment would be ruined. Her aide asked if it was to be -40 Fahrenheit or -40 Celsius. Professor Butler replied, "Whichever one you feel like." Why was she so cavalier about it?

108. A farmer had a 40 pound rock that he used to measure out grain that he sold in 40 pound bags. One day he dropped the rock and it broke into four unequal pieces. At first he was distraught, but then he realized that this was a wonderful turn of events. Using his balance scale and these four pieces he could now measure out grain for any (whole number) poundage between 1 and 40 pounds. What were the weights of the four pieces of rock?

109. People are smarter than computers. My expensive computer algebra system can't do the following problem, but you can. The units digit of a number is its rightmost digit. For example, the units digit of 1776 is 6. What's the units digit of $7^{1808200818082008}$?

110. Here's an oldie, but goodie. How old? It was found on a Babylonian clay tablet dating to about 300 B.C. There are two fields whose total area is 1800 square yards. One field produces 2/3 of a bushel of grain per square yard, while the other produces 1/2 a bushel per square yard. If the total yield is 1100 bushels, what is the size of each field?

111. What is the largest number that can be obtained as a product of positive integers that add up to 100? The correct answer would be $50 \times 50 = 2500$ if only two positive integers were allowed, but there is no restriction on how many, provided they all add up to 100.

112. What would the answer to the previous question be if both positive and negative integers were allowed?

The 200 Years/200 Questions contest is being offered by the Department of Mathematics and Computer Science at Mount Saint Mary's. Seven new problems will be posted each week until we reach 200 at the end of the spring semester. The contest is open to the entire Mount community. Visit http://faculty.msmary.edu/heinold/contest/ for details on prizes.

200 Years/200 Questions — Week 17

Do as many as you like. Send solutions to contest@msmary.edu by Friday February 8.

113. Find the next two letters in this sequence: `B, C, N, O, F, N, E, N, A, M, G`. (Hint: It's not mathematical.)

114. Professor Portier is staying at a hotel and runs into some TV trouble. He tries to turn the TV on with the remote, but it doesn't work. So he walks over to the TV and presses the power button on the set, but again, nothing. Thinking that maybe the maid had knocked the plug loose while vacuuming, he unplugs the TV and plugs it back in, but still no power. Finally, he tries punching the TV because sometimes things really do work after you hit them! But even that doesn't help. So he gives up and calls the front desk. A technician soon shows up and looks at the TV for a bit. He asks Professor Portier to turn on the lamp next to the TV. Professor Portier does so, and the lamp lights up the room. The technician then fixes the problem without using any tools and without unplugging the TV. What did he do?

115. Find nine different 6-digit numbers which are all divisible by 7, 11, and 13. (There's a really quick way to do this.)

116. A boss wanted to give each employee a bonus of \$50. However, the bonus fund was \$5 short. There was only enough to give \$50 to every employee except one who would have to get \$45. This didn't seem fair, so the boss decided to give each person \$45 and keep the remaining money, which totalled \$95, in the fund. How much money was in the fund to start?

117. At one point or another, we've all seen the equation $3^2 + 4^2 = 5^2$. This is associated (via the Pythagorean theorem) with the famous 3-4-5 triangle. There's lots of other triangles like this. For example, there's a 5-12-13 triangle, where $5^2 + 12^2 = 13^2$. Can you find an a-b-c triangle (where $a^2 + b^2 = c^2$) in which at least one of a, b and c is greater than 1000?

118. Professor Heinold has a cute calendar on his desk. It is made up of two cubes with numbers painted on them, and the cubes sit in a lovely container. Each cube has a single digit on each of its six faces and the two cubes together tell which day of the month it is. For instance, on February 4th, one of the cubes is rotated so that the front face shows a 0 and the other is rotated so that the front face shows a 4. The question: If you were designing such a calendar, what digits would you paint on each of the faces of the cubes so that they could display any date from 01 to 31? (Note: It may seem like 12 faces aren't enough, but if you're clever about it.)

119. Find the smallest positive integer that satisfies the following property: If you take the leftmost digit and move it all the way to the right, the number thus obtained is exactly 3.5 times larger than the original number. For instance, if we start with 2958 and move the 2 all the way to the right, we get 9582, which is roughly 3.2 times the original number. Be warned, the smallest positive integer with this property is not all that small.

200 Years/200 Questions — Week 18

Do as many as you like. Send solutions to contest@msmary.edu by Friday February 15.

120. Professor Jarvis stopped at the store on her way home from school and spent \$13.59. She handed the cashier a 50-dollar bill. The cashier, as he handed back the change, exclaimed, "Wow! Would you look at that!" Professor Jarvis was equally surprised. What was so interesting?

121. There is one country missing from the following list. What is it and why?

    `Uganda, Gabon, Indonesia, Sao Tome and Principe, Somalia, Republic of the Congo, Democratic Republic of the Congo, Kiribati, Kenya, Colombia, Maldives, Brazil`

122. Replace the question mark with an integer that fits the pattern:

    `12, i, f, 3, f, y, ?, y, m, 5280, f, m`

123. Professor Petrelli stopped at the market the other day. He bought some apples for 25 cents apiece, some grapefruits for a dollar apiece, and a few exotic dragonfruits at \$15 apiece. He bought at least one of each, and bought nothing else, spending a total of \$100 on exactly 100 pieces of fruit. How many of each did he buy?

124. Professor Heinold has a large drawer full of socks. It contains 100 blue socks, 80 tan socks, 60 black socks, and 40 orange polka-dotted socks. The other morning he was packing socks for a trip and he needed 10 pair. It was dark in the room and he just randomly pulled socks out of the drawer without knowing what color they were. What was the minimum number of socks he needed to remove from the drawer in order to be sure he had at least 10 pair? (A pair consists of two socks of the same color, and assume that no sock can be counted twice.)

125. Find a three-digit number which is equal to the sum of the cubes of its digits.

126. Suppose that your calculator, your computer, and for that matter all your friends' and relatives' calculators and computers are all broken. Suppose further that you have no way of estimating square roots by hand, either by the old methods they used to teach in school or by things you may have learned in calculus. Using only a little algebra and the fact that $\sqrt{169} = 13$, how could you determine which of $\sqrt{10} + \sqrt{17}$ and $\sqrt{53}$ is larger?

The 200 Years/200 Questions contest is being offered by the Department of Mathematics and Computer Science at Mount Saint Mary's. Seven new problems will be posted each week until we reach 200 at the end of the spring semester. The contest is open to the entire Mount community. Visit http://faculty.msmary.edu/heinold/contest/ for details on prizes.

200 Years/200 Questions — Week 19

Do as many as you like. Send solutions to contest@msmary.edu by Friday February 15.

127. The year 1961 reads the same upside down as right-side up. How many times in the last 2008 years has this happened?

128. What do the following words all have in common?

    `mischief, parliament, herd, murder, skulk, bevy, gaze, crash, leap, pride`

129. What was the most recent year whose name in Roman numerals used each numeral once and only once?

130. Professor Heinold earned grades of 95%, 89%, and 83% on his first three arithmetic tests this semester. Only the final exam remains. If the final counts for twice as much as each of the tests, and there are no other grades in the class, what does he need to get on the final to pull off an A (93%) for the semester?

131. Instead of representing the number 1 as a number, suppose we represent it by any letter in its name. For instance, 1 can be represented by o, n, or e. The number 2 can be represented by t, w, or o. Starting with 1, how many consecutive integers can you represent this way so that no two integers have the same representative? In your answer please tell which letters represent which numbers.

132. Give an argument describing why your answer to the previous question is the the best possible. So, if your answer to the last question was 5, show why it would be impossible to represent 1 through 6.

133. A *permutation* of the integers $1, 2, \dots, n$ is any arrangement of all of the numbers with each one appearing exactly once. For example, $(1, 2, 3, 4, 5)$, $(2, 1, 3, 5, 4)$ and $(5, 4, 3, 2, 1)$ are permutations of $1, 2, 3, 4, 5$. There are 120 possible permutations in total for $n = 5$, and in general there are $n!$.

    How many permutations, $(s_1, s_2, \dots, s_n)$, of the integers $1, 2, \dots, n$ have the property that $s_k \geq k - 2$ for every $k = 1, 2, \dots, n$? For instance, of the three examples above, the first two have this property, but the last one doesn't because $s_5 = 1$, and this is not greater than $5 - 2 = 3$.

The 200 Years/200 Questions contest is being offered by the Department of Mathematics and Computer Science at Mount Saint Mary's. Seven new problems will be posted each week until we reach 200 at the end of the spring semester. The contest is open to the entire Mount community. Visit http://faculty.msmary.edu/heinold/contest/ for details on prizes.

200 Years/200 Questions — Week 20

Do as many as you like. Send solutions to contest@msmary.edu by Monday, March 3.

134. The following list of letters has something to do with geography. What is it?

     `A, AA, B, B, C, C, C, C, C, D, F, G, H, H, K, M, PG, QA, SM, S, T, W, W, W`

135. A snail falls to the bottom of a forty-foot well. It can crawl up the sides of the well at a rate of four feet each day, but each night it slides back three feet. At this rate, how long will it take the snail to climb out of the well?

136. In a group of five friends, the sums of the ages of each group of four of them are 124, 128, 130, 136, and 142. What is the age of the youngest?

137. Adding certain numbers to their reversals sometimes produces a number that reads the same backward and forward (a palindrome). For instance, $241 + 142 = 383$. Sometimes, we have to repeat the process. For instance, $84 + 48 = 132$ and $132 + 231 = 363$. Find a two-digit number for which this process must be repeated more than ten times to obtain a palindrome.

138. Thanks to Dr. Bill Collinge for the following problem. The current pay period for the college's employees is 28 days, Feb. 15-March 14 (inclusive). The pay period begins on payday and ends the day before the next payday. Payday is determined as follows:

     - payday is the 15th normally
     - payday is the 14th when the 15th is Saturday
     - payday is the 13th when the 15th is a Sunday
     - payday is the 12th when the 15th is a Monday national holiday.

     Can there ever be a 27-day pay period? Find it or explain why it can't happen.

139. A photographer wishes to arrange ten people into two rows of five. Each person in the back row must be taller than the person directly in front of him/her. Also, the photographer wants the heights to increase from left to right within each row. Assuming everyone has a different height, how many arrangements are possible?

140. The Fibonacci numbers are one of the most famous sequences in all of mathematics. The first two Fibonacci numbers are both equal to 1, and thereafter each Fibonacci number is obtained by summing the previous two numbers. The third number in the sequence is $1 + 1 = 2$; the number after that is $1 + 2 = 3$; and the number after that is $2 + 3 = 5$. The first several Fibonacci numbers are 1, 1, 2, 3, 5, 8, 13, 21, 34, and 55.

     Show that if there's even one Fibonacci number that's divisible by an integer n, then there are, in fact, an infinite number of them that are divisible by n.

200 Years/200 Questions — Week 21

Do as many as you like. Send solutions to contest@msmary.edu by Friday, March 14.

141. 4 is the only number in English with a certain property. Give any number in another language that also has this property. (Please explain your choice.)

142. On *The Price is Right*, one of the games is called Make Your Move. There are three prizes. The contestant is shown a string of nine digits. The goal is to break the string into three parts—a 2-digit number, a 3-digit number, and a 4-digit number—corresponding to the prices of the three prizes (not necessarily in that order). None of the prices overlap; all nine digits are used. Assuming the contestant randomly splits the string into three legal numbers, what are the chances of winning?

143. Let's suppose I am thinking of a number and you have to guess my number. You get to ask me twenty "yes" or "no" questions about the number. What's the largest number I should be allowed to choose so that it is possible for you to guess it based on my answers to your twenty questions?

144. Another puzzle that's too much for most calculators: Find the *exact* value of 1000000000000000004 $\times$ 999999999999999996. Please write the entire number out. (Hint: There's a short way to do this.)

Sequences: A sequence is a list of numbers. Sequences are often given by formulas. For instance, the formula $f(n) = n^2$ generates the sequence $1, 4, 9, 16, 25, \ldots$, and the formula $f(n) = 2^n$ generates the sequence $2, 4, 8, 16, 32, \ldots$. The first entry in the sequence always corresponds to $n = 1$ in the formula, the second to $n = 2$, etc.

For the following three questions, find a simple formula that generates the given sequence. Your answers may not have cases. For example, in the second problem the answer "$f(n) = 0$ if n is odd and 1808 if n is even" consists of two cases and is not allowed. Your answer should be a formula.

145. $1, 11, 111, 1111, 11111, \ldots$

146. $0, 1808, 0, 1808, 0, 1808, \ldots$

147. $1, 1, 2, 3, 3, 4, 5, 5, 6, 7, 7, 8, \ldots$

200 Years/200 Questions — Week 22

Do as many as you like. Send solutions to contest@msmary.edu by Friday, March 21.

148. There is at least one very common English word that has a fairly strange plural—the noun and its plural share no letters in common! What is this noun and its somewhat archaic plural?

149. What year between 1 and 2008 has the longest name when written in Roman numerals?

150. What do the following words have in common?

     `acidic, asks, brewer, igniting, petite, prefer, redivide, teepee, zoo`

151. Give the shortest expression you can that is equivalent to the following: $(x-a)(x-b)\dots(x-z)$. (Hint: Don't work too hard on this one.)

152. A strange way to break up a square root: Assign numbers to the letters A, T, O, and M to make the equation $\sqrt{ATOM} = A + TO + M$ true. Assume that ATOM is a four-digit number and TO is a two digit number.

153. In Professor Heinold's Linear Algebra class students learn several ways to compute the determinant of a matrix. One of those is good for small matrices, but pretty slow for larger matrices. We estimated that it would take 25! (25 factorial) computations to compute the determinant of a 25×25 matrix. That's too much for a person to do, but what about a supercomputer? The fastest supercomputers can do about a trillion computations a second. If 1000 of the fastest supercomputers worked round the clock on this computation, how many years would it take to finish?

154. At the American Crossword Puzzle Tournament (where Professor Weiss came in 10th this year!), solvers compete in one of five skill divisions A, B, C, D, or E. A certain game had people divide into three-person teams according to the following rules:

     - A team can have at most one A-level solver.
     - A team can have at most one B-level solver.
     - If a team has both an A and a B, the third person must be either a D or E.

     How many different combinations of teams are there?

The 200 Years/200 Questions contest is being offered by the Department of Mathematics and Computer Science at Mount Saint Mary's. Seven new problems will be posted each week until we reach 200 at the end of the spring semester. The contest is open to the entire Mount community. Visit http://faculty.msmary.edu/heinold/contest/ for details on prizes.

200 Years/200 Questions — Week 23

Do as many as you like. Send solutions to contest@msmary.edu by Friday, March 28.

155. In English there are many ways of spelling the same sound. For instance, the long $\overline{a}$ sound occurs in *hey, way, make, great, weight*, etc. There are at least 14 different consonant-vowel combinations in English that give the *sh* sound. Can you find seven of them?

156. A variation of the following problem has recently been making the email rounds. I decided to change it by adding some spiders! There are 7 girls in a bus. Each girl has 7 backpacks. In each backpack, there are 7 cats. For every cat there are 7 spiders. How many legs are there in the bus?

157. The previous problem is similar to an old riddle:

    As I was going to St Ives
    I met a man with seven wives
    And every wife had seven sacks
    And every sack had seven cats
    And every cat had seven kits
    Kits, cats, sacks, wives
    How many were going to St Ives?

158. We very rarely write out the names of large numbers in English, and for good reason. The name of the number 123,456, for instance, is *one hundred twenty-three thousand four hundred fifty-six* (that's 48 letters, plus a few hyphens). What number between 1 and 1808 has the most letters in its name?

159. Using each of the digits 0 through 9 exactly once, replace each letter with a digit to make the following statement true: $abcde \div vwxyz = 9$.

160. Suppose there are a bunch of pennies on a table in front of you. Exactly 10 of them are heads-up, and the rest are tails-up. Suppose you have absolutely no way to tell which are which (maybe you're blindfolded and wearing mittens). You are allowed to move these pennies around, lift them up, whatever, but you'll never be able to tell which are heads-up and which are tails-up. Believe it or not, it's still possible to rearrange these pennies into two groups, each having the same number of heads-up pennies. How?

161. Compute $3 + 33 + 333 + \cdots + \underbrace{333\cdots3}_{n}$. Your answer should be a short formula involving n.

200 Years/200 Questions — Week 24

Do as many as you like. Send solutions to contest@msmary.edu by Friday, April 4.

162. The word *part* has the interesting property that if you remove its letters one by one, each resulting step is a real word. For instance, *part* $\rightarrow$ *pat* $\rightarrow$ *pa* $\rightarrow$ *a*. You may remove the letters in any order, and the last word needs to be a real word as well. What's the longest word you can find with this property? (Seven letters and over will count as a correct answer.)

163. A survey of a group of people indicated there were 25 with brown eyes and 15 with black hair. If 10 people had both brown eyes and black hair and 23 people had neither, how many people were interviewed?

164. Let's suppose that you have cutouts of the numbers 2, 3, 4, 5, as well as cutouts of a plus sign and an equals sign. How can you arrange these six cutouts to make a true statement? You may not use anything other than these six cutouts. (Note: placing two numbers next to each other creates a two-digit number.)

165. Here's a problem similar to the previous one, but with an additional trick. You now have five cutouts: the numbers 2, 4, 7, and 6 and the equals sign. Using only these five cutouts, how can you arrange them to make a true statement?

166. Among its many interesting properties, the number 1 is a perfect square, a perfect cube, and a perfect fifth power. What is the next smallest integer that has this property?

167. Professor Butler asked her students to find the largest perfect square that satisfies a certain mathematical property. One student gave the unfortunately long answer of 1,345,036,398,102,485,559,122. Without hesitation, Professor Butler replied, "Sorry. That can't be right. It's not a perfect square." How could she have possibly known so quickly?

168. There are five hats—three red and two white. Three people, Alice, Bob, and Carol, are wearing hats. The other hats are not in view. Alice, Bob, and Carol do not know the color of their hats, but each can see the color of the hats worn by the other two, and they know that there are three red hats and two white hats in total.

     Alice is the first to speak. She says, "I do not know the color of my hat." Bob then adds, "Nor do I know the color of my hat." Carol, having heard these two statements, boldly asserts, "I now know the color of my hat. It is . . . " What color did she name? (As you have a 50-50 shot, please provide the reasoning.)

The 200 Years/200 Questions contest is being offered by the Department of Mathematics and Computer Science at Mount Saint Mary's. Seven new problems will be posted each week until we reach 200 at the end of the spring semester. The contest is open to the entire Mount community. Visit http://faculty.msmary.edu/heinold/contest/ for details on prizes.

200 Years/200 Questions — Week 25

Do as many as you like. Send solutions to contest@msmary.edu by Friday, April 11.

169. It is possible to insert a space into a seven-letter math word to make two words that could be used to describe a man who has just spent a long day at the beach. What's the math word?

170. Which president's mother did not vote for her son for president? Here are your three choices: JFK, William McKinley, and Jimmy Carter. Please explain your choice. (Note: each woman was alive at the time her son was elected president).

171. Alicia Agnew sends along this riddle: "You have but to speak my name to break me." Can you find the answer?

172. One laser blast will break asteroids larger than 20 kg into three pieces, each one third of the mass of the original. Asteroids smaller than 20 kg are shattered into dust by the laser. How many laser blasts would be required to reduce a 2000 kg asteroid to dust?

173. How many different seven-digit numbers can be made by rearranging the digits of 3053354? (Note that this includes the given number, and the first digit of a number is never 0.)

174. The number $2^{10} = 1024$ is four digits long. How many digits long is $2^{18082008}$? (Hint: There's a certain key on almost any scientific calculator that is very helpful here. However, entering $2^{18082008}$ will surely produce an overflow error, so you have to be a little tricky.)

175. A shifty man walks into a mathematics warehouse store to buy a \$20 calculator. The warehouse makes a hefty profit on these calculators; it buys them for \$6 apiece from the manufacturer. The man hands the clerk a hundred dollar bill. The clerk, not having change for a hundred so early in the morning, heads over to the donut shop across the street to ask for change of a hundred. The donut shop has the change, but the cashiers don't want to give up all of their twenties. Just then, a customer pays for his (huge order of) donuts with a wad of twenties and the manager gives the clerk five of those twenties in exchange for the hundred. The clerk returns and gives the shifty man his change. Later that day, the donut shop manager comes over to the warehouse angry because the hundred turned out to be counterfeit. Embarrassed, the clerk hands her two fifties from the register. Here's the question: How much money was lost due to the counterfeit hundred, and who lost it?

The 200 Years/200 Questions contest is being offered by the Department of Mathematics and Computer Science at Mount Saint Mary's. Seven new problems will be posted each week until we reach 200 at the end of the spring semester. The contest is open to the entire Mount community. Visit http://faculty.msmary.edu/heinold/contest/ for details on prizes.

200 Years/200 Questions — Week 26

Do as many as you like. Send solutions to contest@msmary.edu by Friday, April 18.

176. A group of math professors rake up three piles of leaves from the front of the Science Building and six piles from the back of the building. When they put all the piles together, how many piles of leaves will they have?

177. In a particular area of France, three out of ten people have telephone numbers that are not listed in the phone book. If you choose 100 names from the directory at random, how many would have unlisted numbers?

178. A baby rabbit has a number of carrots in his cage. Each day he eats one quarter of the carrots. After four days he has eaten 350 carrots. How many were there to start with?

179. How many of the numbers between 1 and 10,000 contain the digit 3?

180. How many integers from 1 to 10^{30} inclusive are not perfect squares, perfect cubes, or perfect fifth powers?

181. How many ordered triples (x, y, z) of positive integers satisfy $xyz = 4000$?

182. How many positive integers, n, are there such that $n^2 - 440$ is a perfect square?

The 200 Years/200 Questions contest is being offered by the Department of Mathematics and Computer Science at Mount Saint Mary's. Seven new problems will be posted each week until we reach 200 at the end of the spring semester. The contest is open to the entire Mount community. Visit http://faculty.msmary.edu/heinold/contest/ for details on prizes.

200 Years/200 Questions — Week 27

Do as many as you like. Send solutions to contest@msmary.edu by Friday, April 25.

183. Hidden inside each of the four words below is another word having a similar meaning. Removing some of the letters and keeping the others in their original order reveals the hidden word. For example, `Joviality` contains the word `Joy`.

     Try these: `Observe, Respite, Instructor, Municipality.`

184. What gets harder to catch the faster you run?

185. The hypotenuse of a right triangle is 26 inches long, and the sum of the other two sides is 34 inches. How long are each of the two sides?

186. Find two real numbers (not necessarily integers) whose product is 200 and whose sum is as large as possible. This problem is a breeze for calculus students, but anyone can do it with a little effort.

187. Recently, on *Wheel of Fortune*, Professor Petrelli won a new $200,000 Ferrari, a $5,000 trip to Belgium, and some cash. He had to pay 20% tax on all of his winnings. Fortunately, the cash he won was exactly enough to pay off all of the taxes. How much cash did he win?

188. Be careful with this one: Professor Petrelli drives his newly won Ferrari around a one-mile circular track at 30 mph. At what speed must he travel on his second lap in order to average 60 mph for the two laps? (The answer is not 90 mph.)

189. Sometimes polynomials have imaginary roots. For instance, $x^2 + 1$ has two imaginary roots, $\pm i$, and $x^2 - 4x + 13$ has two imaginary roots, $2 \pm 3i$. The polynomial $x^3 + ax + 1$ always has at least one real root, and depending on what a is, the other roots may be real or imaginary. For what values of a does $x^3 + ax + 1$ have imaginary roots? Please provide the exact answer. (Hint: If $x^3 + ax + 1$ has imaginary roots, what must its graph look like?)

The 200 Years/200 Questions contest is being offered by the Department of Mathematics and Computer Science at Mount Saint Mary's. Seven new problems will be posted each week until we reach 200 at the end of the spring semester. The contest is open to the entire Mount community. Visit http://faculty.msmary.edu/heinold/contest/ for details on prizes.

200 Years/200 Questions — Week 28

Do as many as you like. Send solutions to contest@msmary.edu by Friday, May 2.

190. Professor Portier was driving to school the other day when he saw the following license plate: `TI-3VOM`. He was a little insulted. Why?

191. What five-letter word, if you remove its first four letters, still sounds the same?

192. Chris Lewis sends in the following problem: Fill in the blanks below with numbers to make the sentence true. Be sure use the number and not its name (i.e., 3 instead of "three").

     `In this sentence, the number of 0's is __, the number of 1's is __, the number of 2's is __, the number of 3's is __, the number of 4's is __, the number of 5's is __, the number of 6's is __, the number of 7's is __, the number of 8's is __ and the number of 9's is __.`

193. The number in each position is equal to the sum of the two numbers directly beneath it. Fill in the pyramid.

```
            ___
        ___     52
    36      ___     36
___     ___     ___     ___
15  ___     ___     ___  11
```

194. A clock in Professor Jarvis's office displays the time and date in the format `HH:MM:SS MM/DD`. The hour is given in the 24-hour format (e.g., 14:00 instead of 2:00pm). At what time and what date of the year does it first happen that all ten digits, 0-9, appear on the display at the same time?

195. The number 99 has the property that if we multiply its digits together and then add the sum of its digits to that, we get back to 99: $9 \times 9 + (9 + 9) = 99$. Find all other positive integers that have this property.

196. You are shown a room containing only a plugged-in lamp holding a single incandescent light bulb. You are led out of the room, and the room is then closed up. You are next shown a bank of three switches, all currently off. One of the three switches controls the lamp; the other two are not connected to anything. Your goal is to figure out which switch controls the lamp. You can flip the switches as much as you like and take as much time as you like. The catch is that once you open the door to the room, the switches will no longer work. (The room is sealed tightly enough that no light can escape through the cracks in the door.) How do you figure out the correct switch?

The 200 Years/200 Questions contest is being offered by the Department of Mathematics and Computer Science at Mount Saint Mary's. Seven new problems will be posted each week until we reach 200 at the end of the spring semester. The contest is open to the entire Mount community. Visit http://faculty.msmary.edu/heinold/contest/ for details on prizes.

200 Years/200 Questions — Last Four Questions

Do as many as you like. Send solutions to contest@msmary.edu by Monday, May 5.

197. Suppose we write all the words in the dictionary backwards and then arrange these backwards words alphabetically. What would be the *last* word in this strange dictionary?

198. In the calculation $43 + 57 = 207$, every digit is precisely one away from its true value. What is the correct calculation?

199. In a magic square, each row, each column, and both diagonals add up to the same number. Can you fill in the missing entries to make the following a magic square?

5	_	_
_	6	2
3	8	_

200. 60 S in a M → "60 seconds in a minute." Try these:

    32 DF at which WF
    29 D in F in a LY
    200 Y since MSM was F by JDuB

The 200 Years/200 Questions contest is being offered by the Department of Mathematics and Computer Science at Mount Saint Mary's. Seven new problems will be posted each week until we reach 200 at the end of the spring semester. The contest is open to the entire Mount community. Visit http://faculty.msmary.edu/heinold/contest/ for details on prizes.