Rubik's Cubes

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A variety of cubes

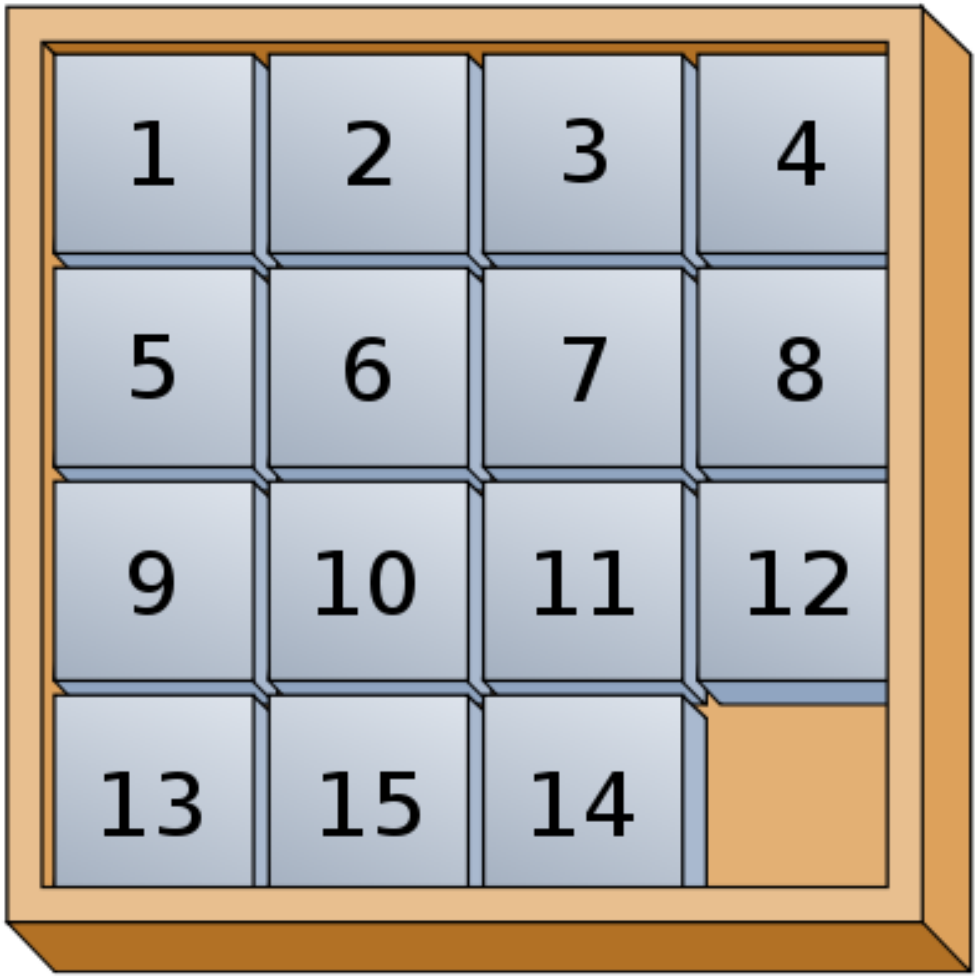
http://en.wikipedia.org/wiki/File:Rubik%27s_Cube_variants.jpg

About the cube

- 1974 Ernő Rubik
- debuted in 1980

http://en.wikipedia.org/wiki/File:Disassembled-rubix-1.jpg

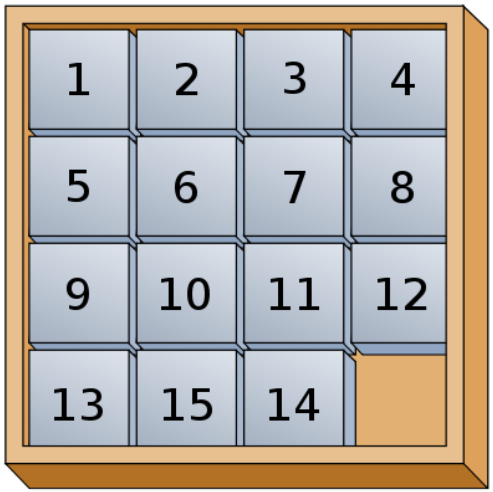
The 15-Puzzle



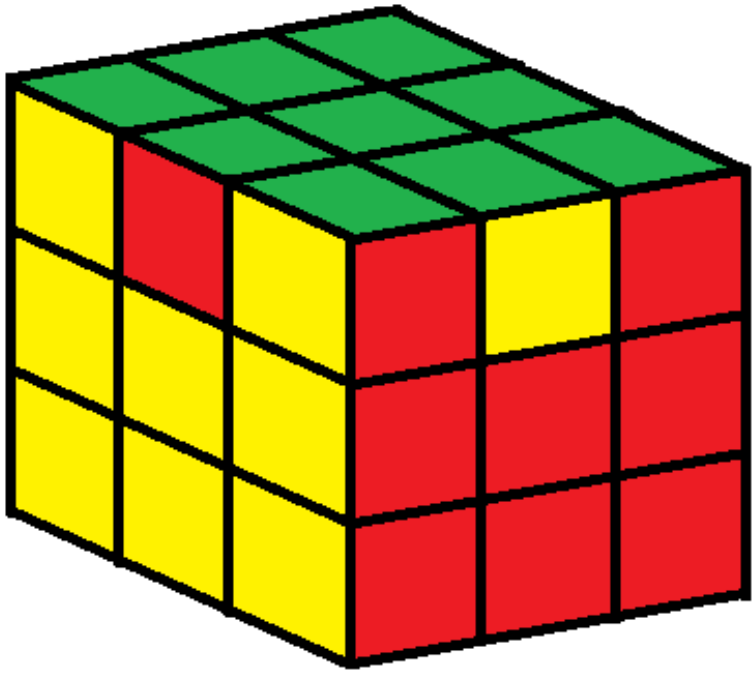
http://en.wikipedia.org/wiki/File:15-puzzle-loyd.svg

Why it can't be solved

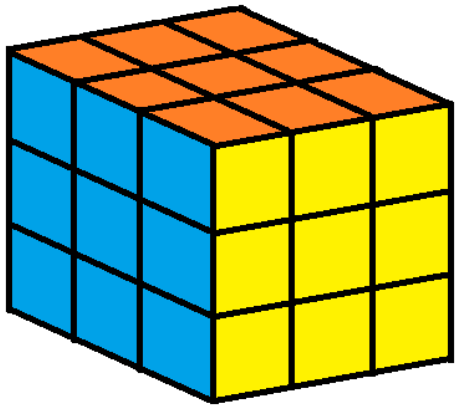
Swapping 14 and 15 is an *odd permutation*.
But it takes an even number of permutations to move the blank square away from and eventually back to the bottom right corner.
This can be made precise using abstract algebra.



Impossible Cube



Counting



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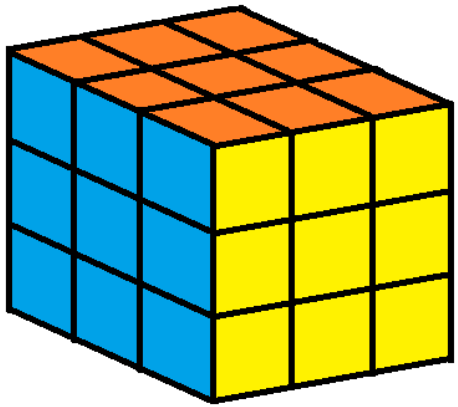

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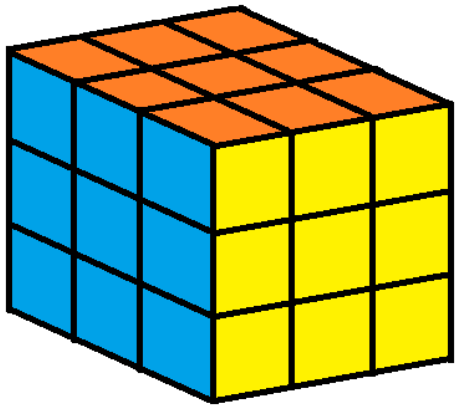

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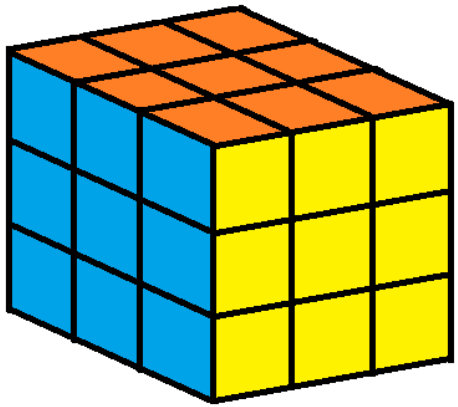

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- If you disassemble and randomly reassemble the cube, there is a 1 in 12 chance that it will be solvable.
- If you peel off the stickers and randomly replace them, the probability that the new cube will be solvable is

$$\frac{8! \cdot 3^7 \cdot 12! \cdot 2^{11}}{54!/(6! \cdot (9!)^6)} \approx 3.1 \times 10^{-16}.$$

Number of moves needed to solve the puzzle

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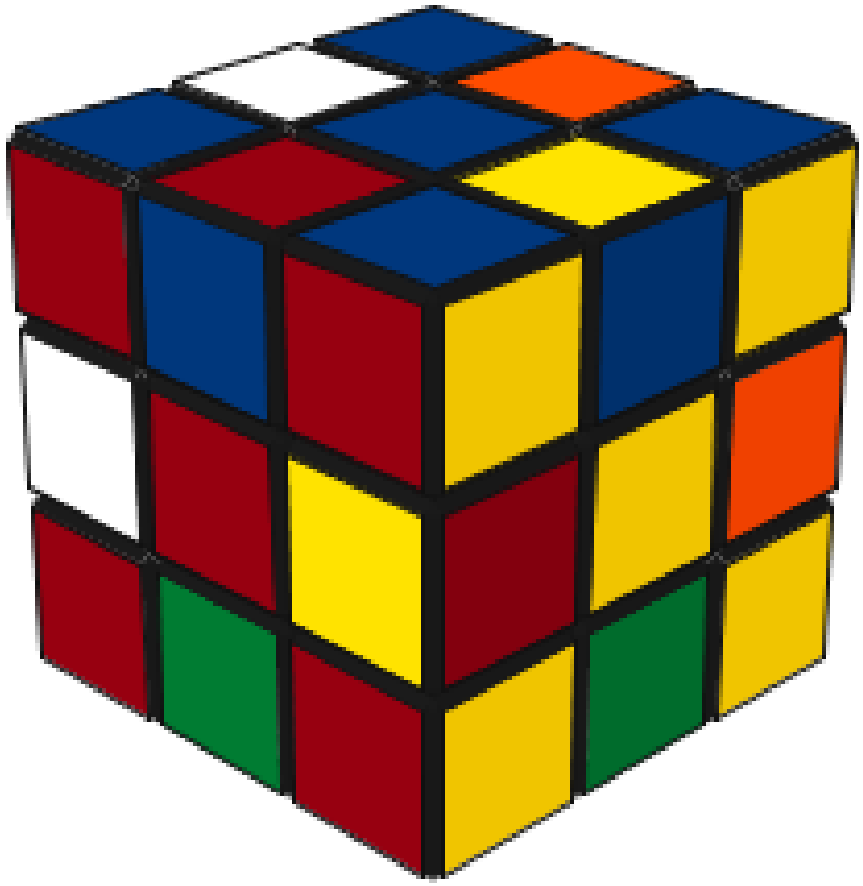
- See www.cube20.org for more.

Superflip



Records

size	record time (seconds)	average
2	0.69	2.12
3	5.66	7.66
4	30	
5	56	
6	115	
7	193	

`http://en.wikipedia.org/wiki/Speedcubing`

World record video

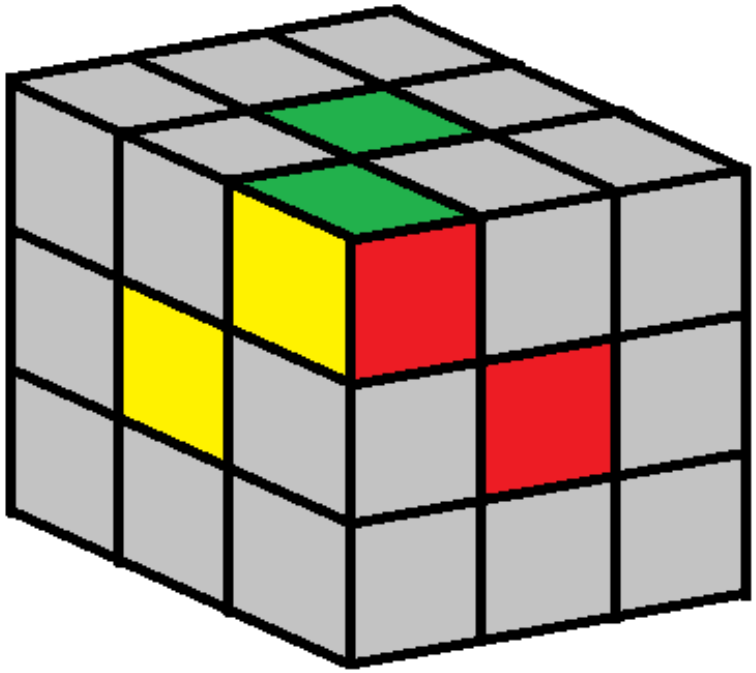
Slow motion

How I learned to solve the cube

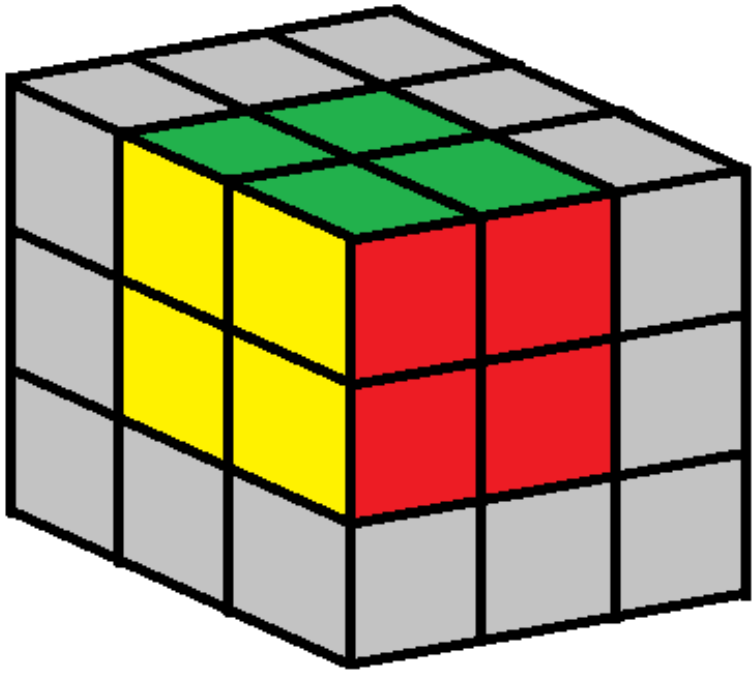
Petrus Method: http://lar5.com/cube/

1. Build a $2 \times 2 \times 2$ corner
2. Expand to $2 \times 2 \times 3$
3. Twist the edges
4. Finish 2 layers
5. Position the corners
6. Twist the corners
7. Position the edges

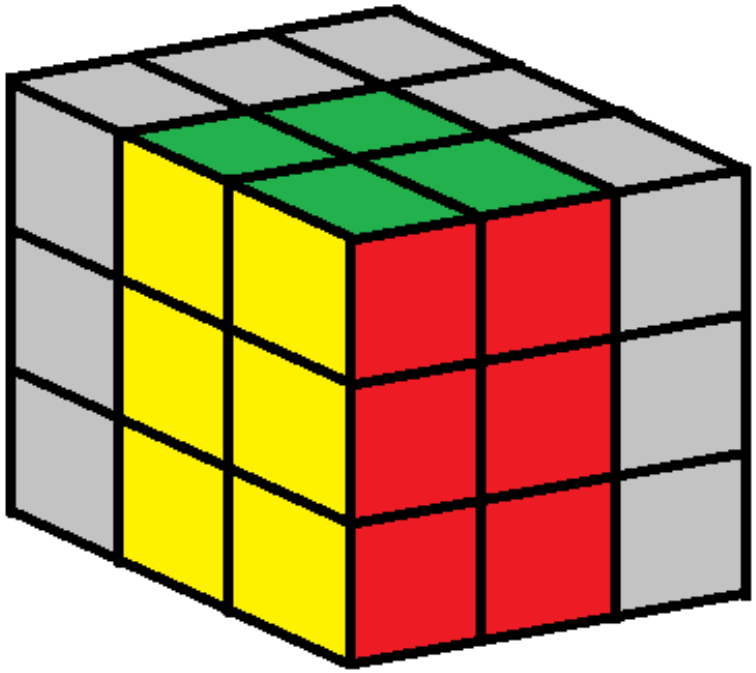
Corners



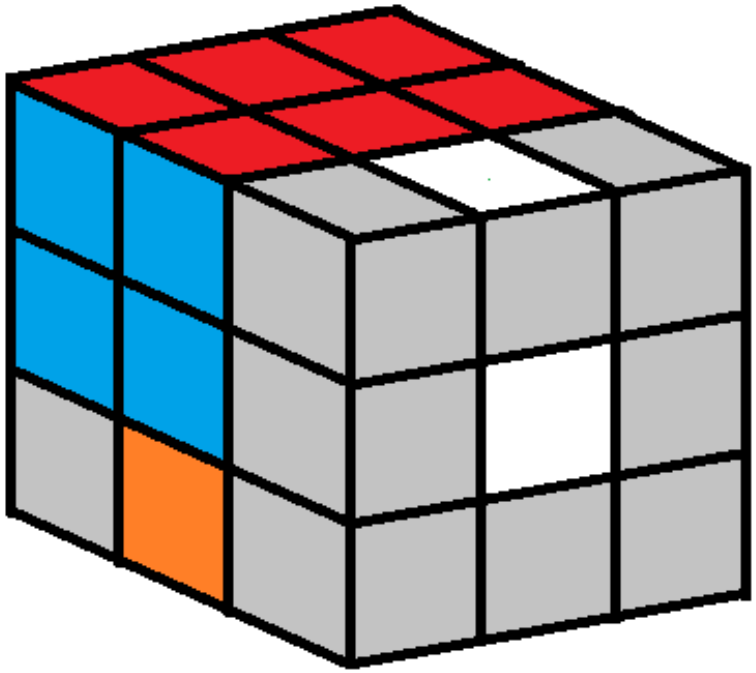
$2 \times 2 \times 2$ Corner



$2 \times 2 \times 3$



“Twist the edges”



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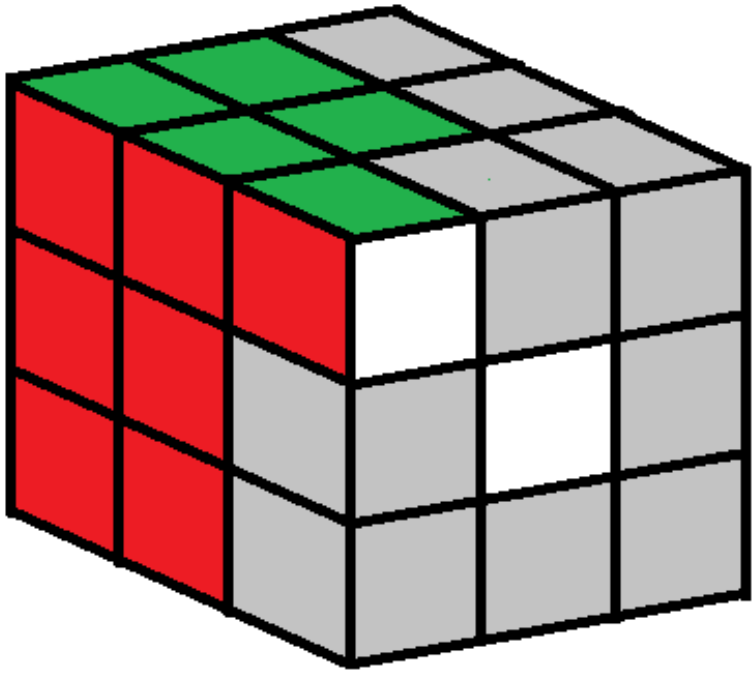

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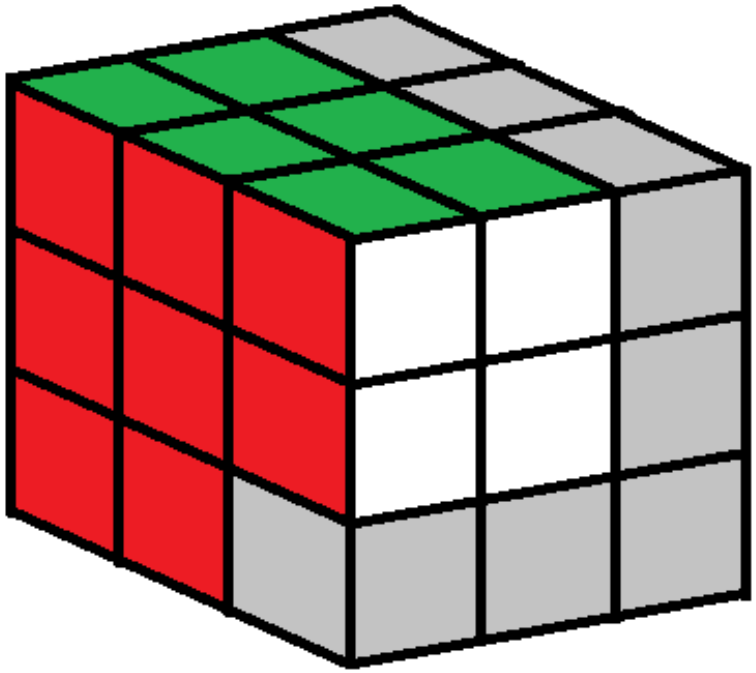

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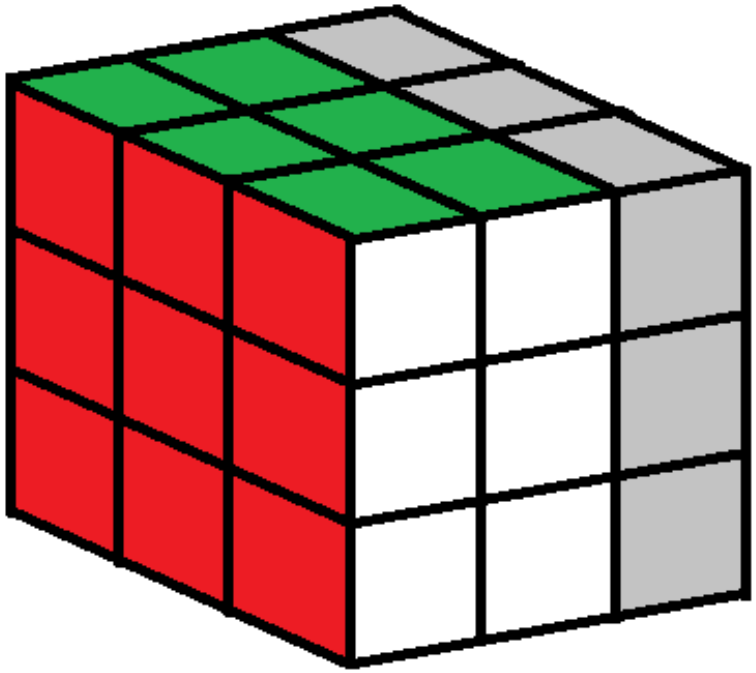
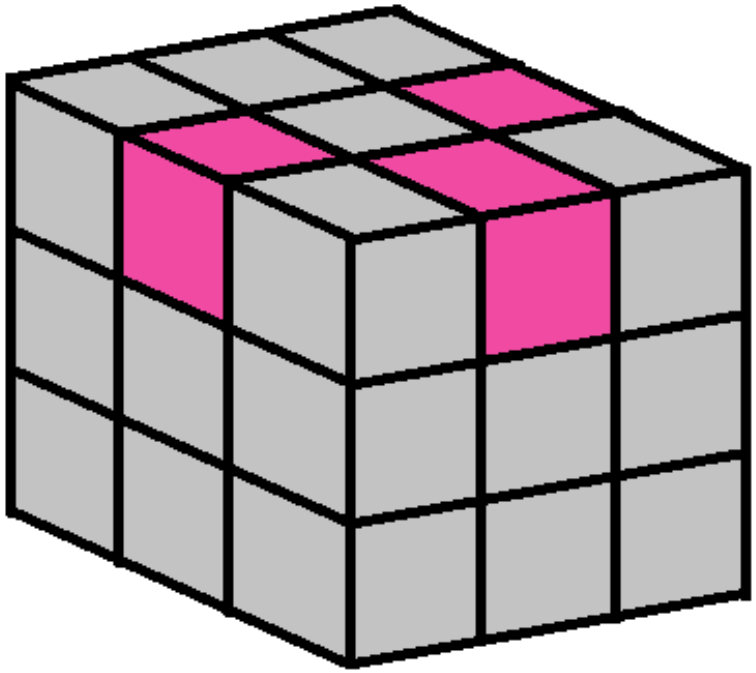
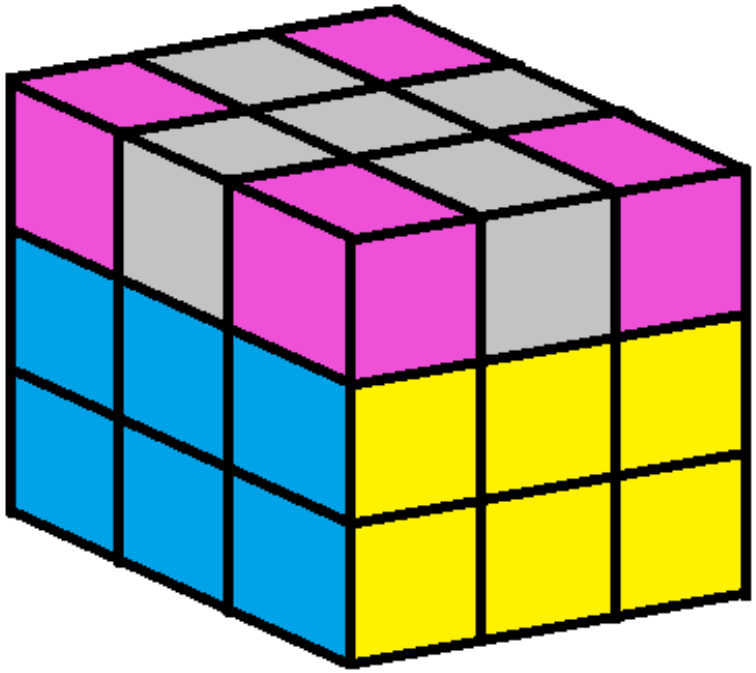


Algorithmic Interlude...



“Position the corners”



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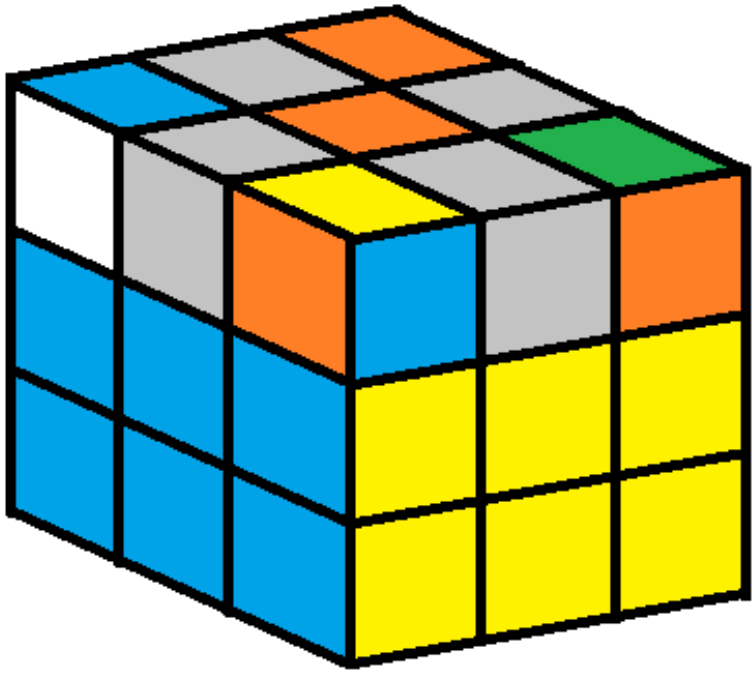

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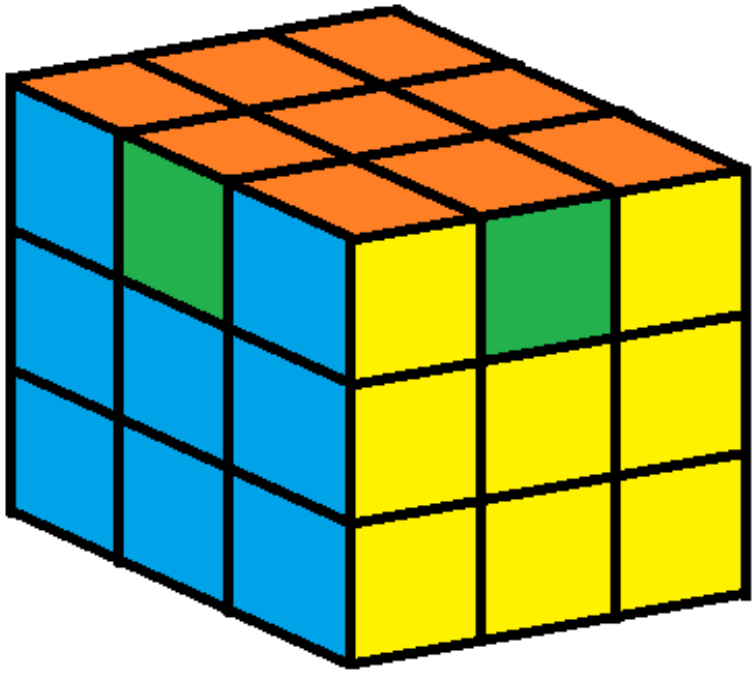
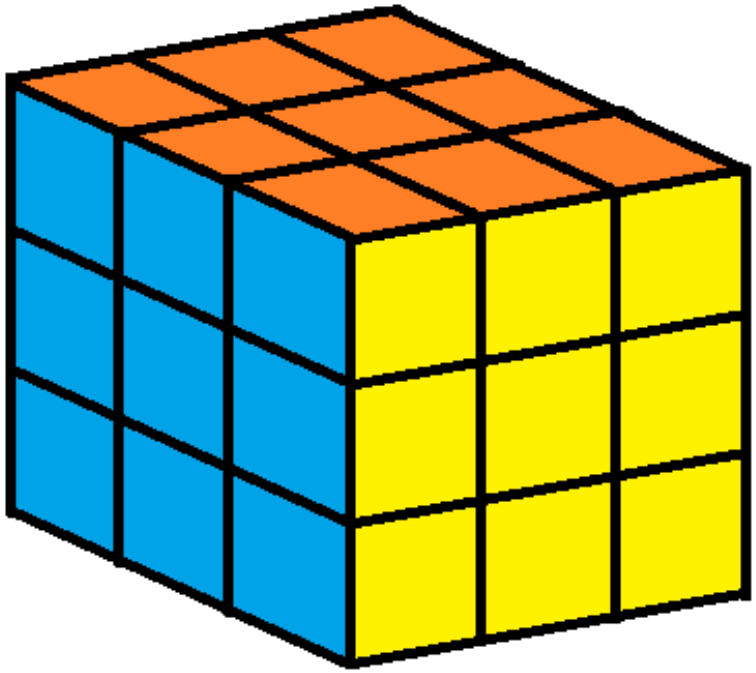


Done!



Sources

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