

Some Original Estimation/Fermi Problems

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- All of these are things we can make reasonable guesses at or quickly look up.

What are Fermi/estimation problems?

- Named for Enrico Fermi who was known for his skill at estimation.
- Goal is to get an answer of the right order of magnitude.
- Usually don't require much more math than some arithmetic and occasionally some unit conversions, percentages, etc.

Why teach this?

- This is the math I use more than anything else in my everyday life.
- It doesn't seem to be taught widely in high school
- Good critical thinking problems
- Fun

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- Common mistakes:
 - Walking too fast
 - No rest
 - Just dividing length by speed
 - Jumping to 20 miles/day with no intermediate steps

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- How:
 - 1 People in US (assuming we're most of their business)
 - 2 Average number of bottles per person (but don't get lost in weeds)
 - 3 How many tomatoes go into a bottle.
 - 4 How much a tomato weighs.

First assignment this spring

- 1 For a typical person who drinks soda, how many pounds of sugar do they end up consuming over the course of a year from that soda?
- 2 How many new cars are sold in the U.S. in a typical year? Estimate this using only basic estimates about people and their car-buying habits.
- 3 Let's say the school president wants to schedule a one-hour meeting over the next year with each undergraduate student here. Will this work? Explain, showing your calculations.

Second assignment this spring

- 1 A student has an idea for a project here called “A Week at the Mount.” Every undergrad will record 10 hours of video of their daily life every day for a full week. All the videos will then be uploaded and stored on hard drives in the library. How much money in hard drive space will this require?
- 2 How many undergraduate classes are there at Mount St. Mary’s this semester? The only thing you need to look up for this is how many undergrads there are.
- 3 If we were to gather all of the world’s population in one place, standing room only, giving each person a comfortable amount of space, how many square miles of space would we need? Give a place that has a comparable area to what you found.
- 4 Suppose you are looking at buying a car for \$25,000. The car’s fuel efficiency is 25 miles per gallon. There is also a \$30,000 model of the same car that is the same except it gets 30 miles per gallon. Purely from a money perspective, does it make sense to buy the \$30,000 model?

Third assignment this spring

- 1 A person lives in a one-room apartment with no heat and plans to heat it with a single, ordinary space heater. How much do you estimate this will cost this calendar year?
- 2 How many people have graduated MSM in its 214-year history? Some notes: graduating classes were no more than a few dozen up until the 1920s. Since then, classes have slowly grown and we now graduate around 400 students a year.
- 3 My office has a bookshelf that takes up an entire wall, floor to ceiling, filled with books (about 8 feet by 8 feet). Students often ask me if I've read all of them. Estimate how long it would take to read all those books.
- 4 A friend wants to start a business plowing out driveways after snowstorms in the Emmitsburg area. Estimate how much money they could expect to earn in a typical year.

This year's test question on first exam

- Suppose you want to take a leisurely road trip from Emmitsburg to California and back. Estimate the total cost of the trip accounting for food, gas, and lodging. Be sure to carefully list all assumptions, show all calculations, and keep your assumptions as simple as possible.

Difficulties I've run into

- Biggest problem: students make very broad assumptions instead of working from first principles.
- The internet knows too much.
- Students not checking for reasonableness of their answers.
- The problems are more open-ended than students are used to.

How I grade them

- I first check if the answer is in the right ballpark.
- Make sure assumptions are right things to be assuming.
- Make sure the values they use for their assumptions are reasonable.
- Check calculations for correctness.

Coming up with problems

- Pay attention during your everyday life for things that might work well.
- Trick is to come up with things where there is actual estimation involved.
- The internet knows too much.

A few other problems

- How many babies were born in the U.S. in 2018?
- How many people turn 75 this year in the U.S.?
- How many people are moving to a new home today?

A few other problems

- Suppose we are planning to hold a concert on one of the Mount's fields. If the field is 2 acres and will be filled with people, how many will there be?
- A few years ago, they replaced all the incandescent lights in the Science Building classrooms with LED lights, which use less electricity. How much does that save over the course of a year?
- If you were to store all the gas you would need for your car over its lifetime in a single tank, how big would it have to be?

A few other problems

- How many words have you spoken in your life?
- How many miles does a soccer player run over the course of a game?
- What does a million dollars look like?

A few other problems

- Congratulations! You've been elected to be the new Frederick County executive. The highway median of Route 15 from Frederick to the PA border used to be mowed by the state but now, as a cost-cutting measure, they want the county to take over mowing it. Estimate how much will this cost the county in labor to mow the median each year. Note: Only estimate the labor cost, not equipment or gas. Also, estimating the number of workers is not really the best approach. It's better to think about the total amount of work involved.
- The Mount actually used to have people do students' laundry up in Borders. Estimate how much it would cost to pay people to do the laundry for all residential students at MSM for a typical school year. Be sure to list your assumptions and show all your calculations.

A few other problems

- The chair of the math department is upset at how much paper I print for this class. Each sheet costs 7 cents to print. Estimate the total cost of all the paper I print for our class over the course of a semester. List out all your assumptions and show your calculations.
- Suppose you were to get paid for doing your schoolwork. Estimate how much you would earn in total over your four years here.
- Before the printing press, if you wanted a new copy of a book, someone literally had to write out a copy by hand. Suppose this was your regular job. Estimate how many books you could copy in a year.

A few other problems

- An earthquake has struck the Mount. The only real damage was that all the books and journals were knocked off the library shelves. The dean of the library decides to hire some student workers at minimum wage to reshelve all the books. Come up with a rough estimate for how much this will cost the library.
- Estimate how many tons of snow you move when shoveling out a four-car driveway from a one-foot snowfall.
- Every day at work, Dr. Heinold spends \$10 buying lunch. Estimate how much he would save over the course of year if he brought his own lunch. (You will have to make a number of assumptions here. I don't care what they are as long as they are reasonable and you clearly indicate what they are.)

A few other problems

- Some people who born and died in the 1800s have grandchildren who are alive today. Could there in fact be people born in the 1700s who have grandchildren who are alive today? What is the upper limit? Of all the people alive today, what's the earliest one of their grandparents could theoretically have been born?
- Over your four years here, you will have to write a bunch of papers for classes. Estimate the total number of words you will end up writing. Please clearly list all your assumptions and show your calculations.